Internet Architecture and Protocols CS 7260

Scribe Notes for 10/19/2012 by Sriram Madapusi Vasudevan

Note:

The following notes will deal with the proof for the Head of Line blocking with input queuing along with saturated throughput to be 2- √2.

Basic Probability Knowledge:

An event is any subset of set of all possible outcomes. Let's say all possible outcomes are contained in Ω, and the event we consider is a particular ω.
Then the event is such that $\{\omega \epsilon \Omega\}$
Two events A_1 and A_2 are considered disjoint when $A_1 \cap A_2 = \emptyset$

Let us consider two events A_1 and A_2 are disjoint.

$P(A_1, A_2) = P(A_1 \cup A_2) = P(A_1) + P(A_2)$

Now, Let us consider events A1 and A2 to be independent

$P(A_1A_2) = P(A_1 \cap A_2) = P(A_1) * P(A_2)$

Assumptions for the Proof:

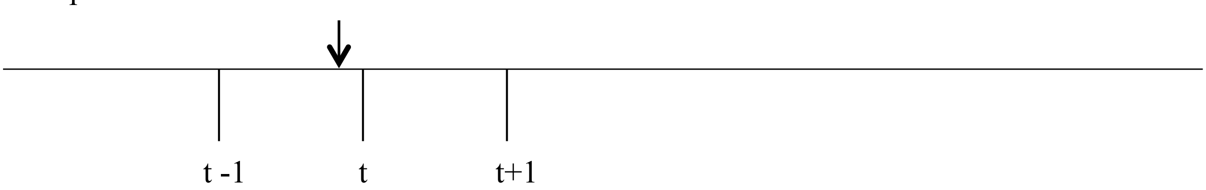
- Each packet that arrives at any input port has equal probability to go to any output port.
- Every packet has equal length.
- Equal size packets are referred to as cells.
- Every cycle is precisely enough time to transfer exactly one packet from input port to output port. (Cycle is the amount of time to transfer a packet).
- Let $\gamma(N)$ be the saturation throughput for a given N * N switch with head of line blocking.
- Receiving no outputs at the output port is only the result of collisions and not because there is no input at this point of time. Since there is saturation throughput there will always be input to the input ports.

Required to Prove:

$$\lim_{N\to\infty} \gamma(N) = 2 - \sqrt{2}$$

Proof:

Since we have considered input saturation, we can say that there will be N Head of Line packets. t - ε



Consider a timeline where switching happens at some time $t - \varepsilon$ and a new packet arrives at a time t.

Now,

$Q_j(t)$ = No. of Head of Line Blocking packet headed for output port j at time $t + \varepsilon$

$Q_j(t+1) = \max \{0, Q_j(t) - 1 + A_j(t+1)\}$

where $A_j(t+1)$ = No. of new cells going to output port j at time t