CS 7643: Deep Learning

Topics:

- Notation + Supervised Learning view
- Linear Classifiers
- Neural Networks
  - Modular Design

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Administrativia

- Now that the dust has settled
  – Nearly everyone who submitted HW0 was added to class from waitlist

- Canvas
  – Anybody not have access?

- HW0
  – People who were on waitlist, your HW0 scores will be uploaded soon.
  – Remember, doesn't count towards final grade.
  – Self-assessment tool.

Plan for Today

- Notation + Supervised Learning view
- Linear Classifiers
- Neural Networks
  - Modular Design

Notation

Vh

Scalars x, y, z, w, a, i

$\vec{x}, \vec{w}$

Sets/Matrices X, Y, W

R.V

d $\vec{x} \in R^d$ $\vec{w} \in R^d$

N

$\theta, \vec{w}$

Raw Input $\vec{x} \in \mathbb{R}^d$

$\in \mathbb{R}^{H \times W \times 3}$

$\in V = \{1, \ldots, d\}$

$\{\text{'a'}, \text{'the'}, \ldots\}$

$\in V^T$

$y \in \mathbb{R}^K$ $\{-1, +1\}$ $\{1, \ldots, K\}$

$f^*: X \rightarrow Y$

$D = \{(\vec{x}_1, y_1), \ldots (x_N, y_N)\}$

$x \sim P^*(X) \quad y \sim P^*(Y \mid X = x)$

$(x, y) \sim P^*(X, Y)$

Goal: Given D

1) Hypothesis space $\mathcal{H} = \{ \underline{h} : X \to Y \}$

Model

2) "Fitness" $L(h ; D) = \frac{1}{N} \sum_{i=1}^{N} L_i(h)$

$L_i = (\underline{h(x_i)}, \underline{y_i^{gt}})$

3) Supervised Learning

$$= \boxed{\underset{\boxed{h \in \mathcal{H}}}{\text{argmin}}} \ \boxed{\text{Loss}}(h ; D)$$

$f^*: X \rightarrow Y$

$D = \{(\vec{x}_1, y_1) \ldots (x_N, y_N)\}$

$x \sim P^*(X) \quad y \sim P^*(Y \mid X = x)$

$(x, y) \sim P^*(X, Y)$

Goal: Given D

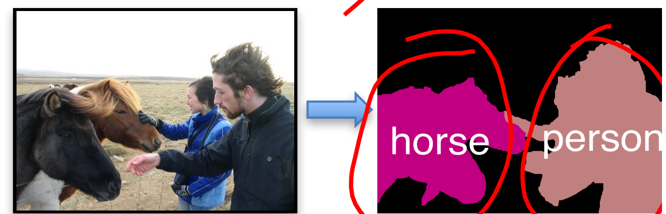
Supervised Learning

- Input: x (images, text, emails…)
- Output: y (spam or non-spam…)
- (Unknown) Target Function
  - $f: X \rightarrow Y$ (the "true" mapping / reality)
- Data
  - $(x_1,y_1), (x_2,y_2), \ldots, (x_N,y_N)$
- Model / Hypothesis Class
  - $h: X \rightarrow Y$
  - $y = h(x) = \text{sign}(w^T x)$
- Learning = Search in hypothesis space
  - Find best h in model class.

Error Decomposition

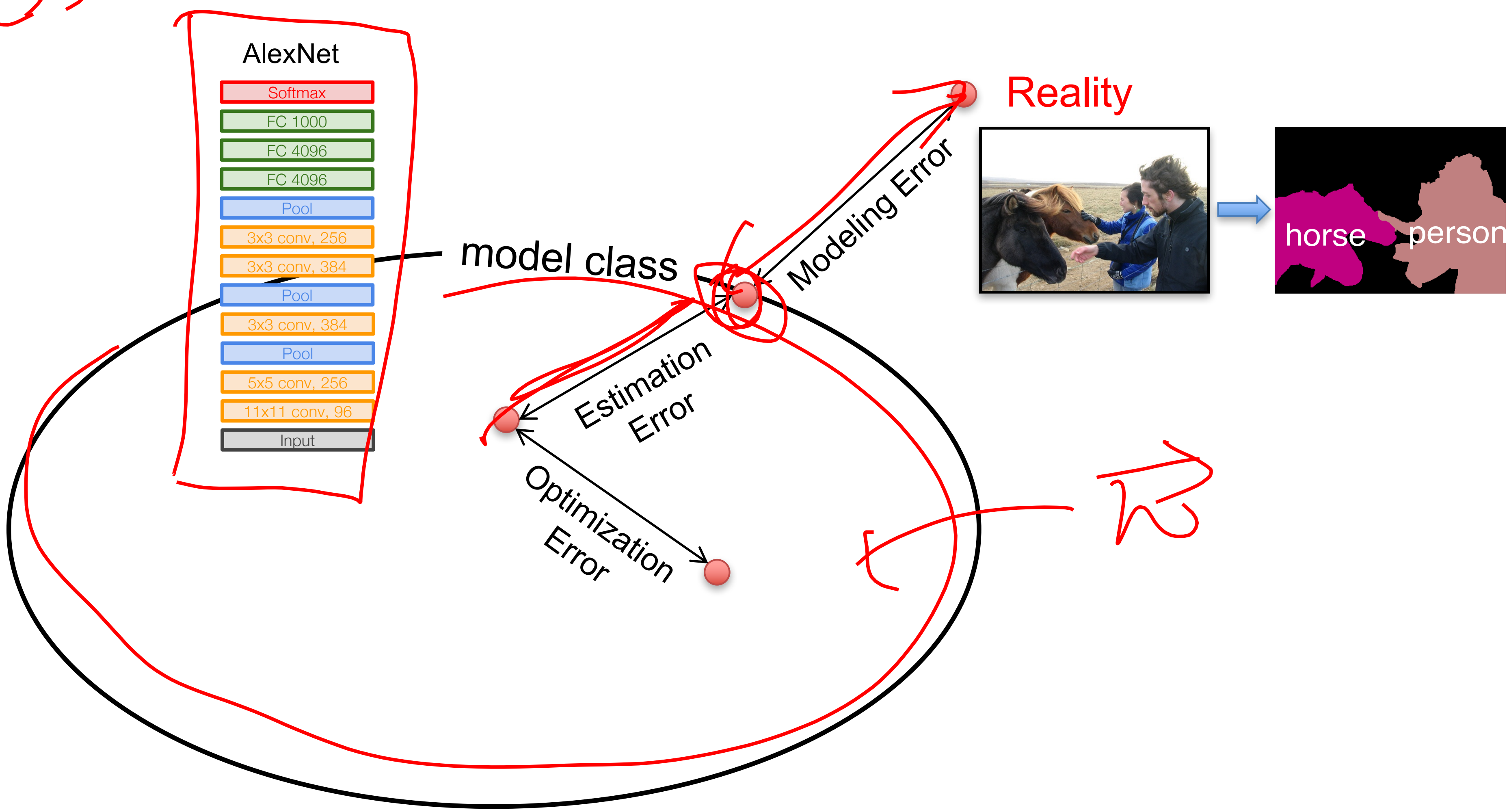
$X \quad f \quad Y$

- Reality

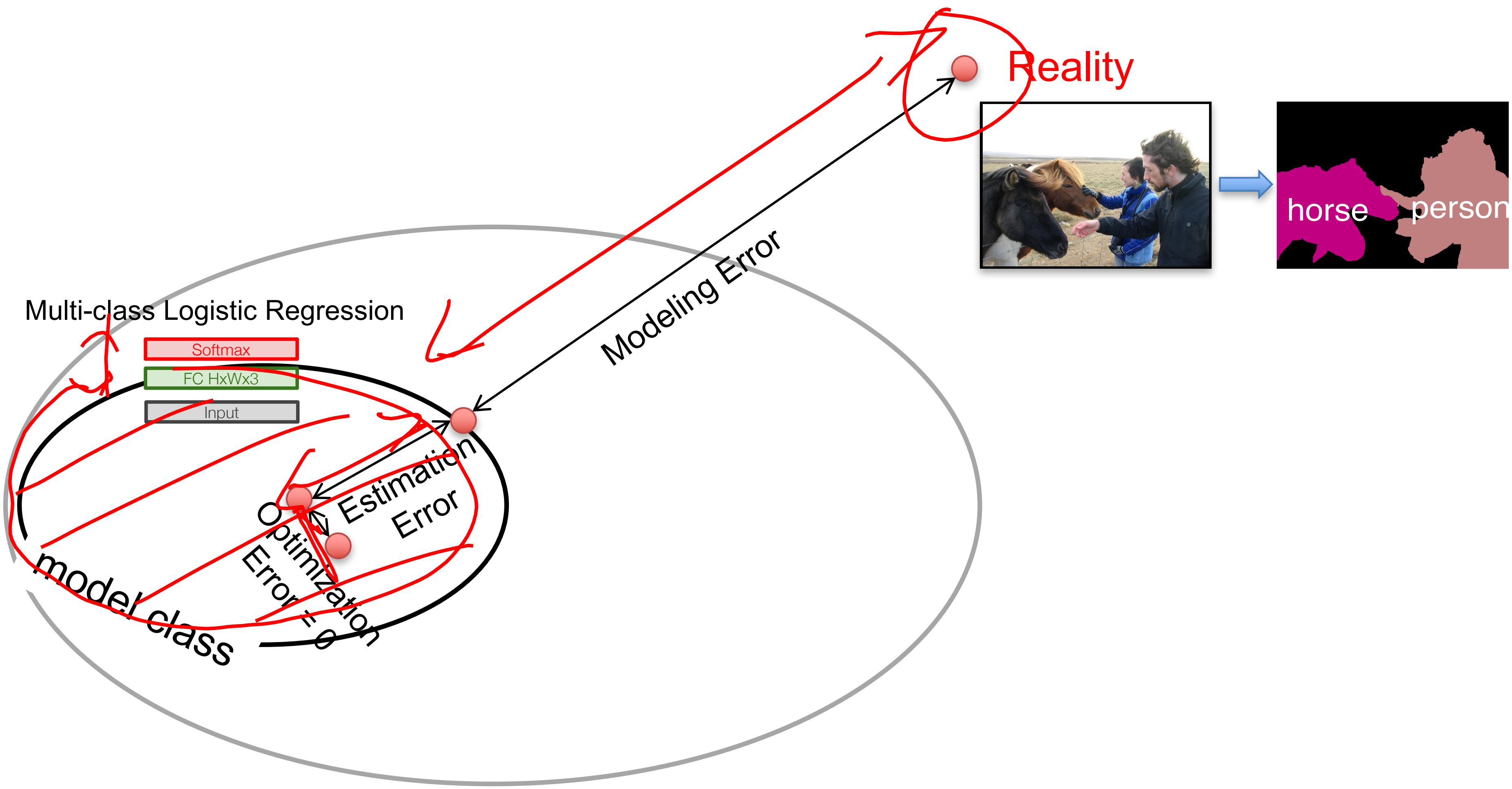


Error Decomposition

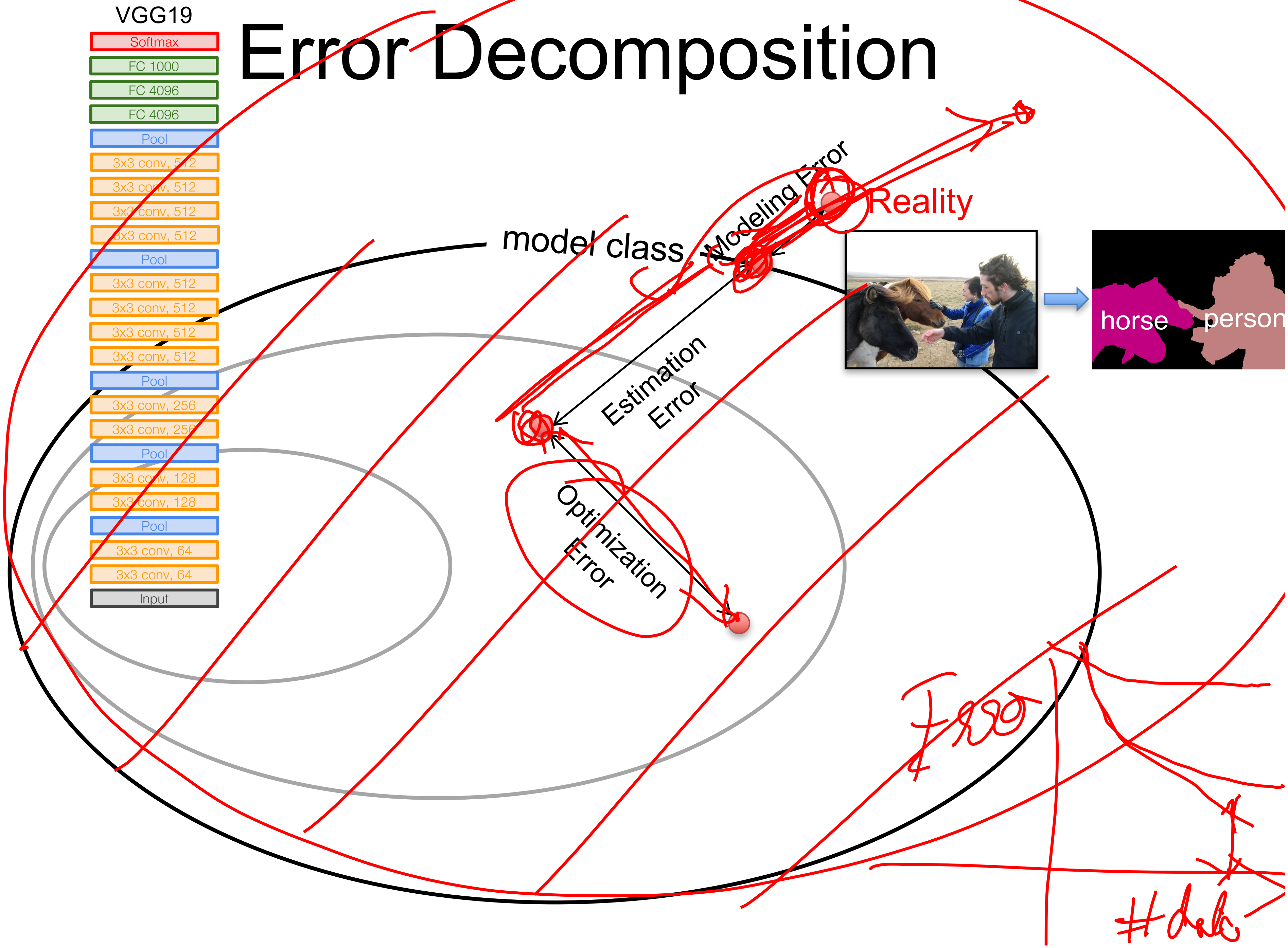
$y = f(\vec{x}; \vec{w})$



Error Decomposition



Error Decomposition



Error Decomposition

- Approximation/Modeling Error
  - You approximated reality with model
- Estimation Error
  - You tried to learn model with finite data
- Optimization Error
  - You were lazy and couldn't/didn't optimize to completion
- Bayes Error
  - Reality just sucks

Linear Classification

Neural Network

Linear classifiers

Image Captioning

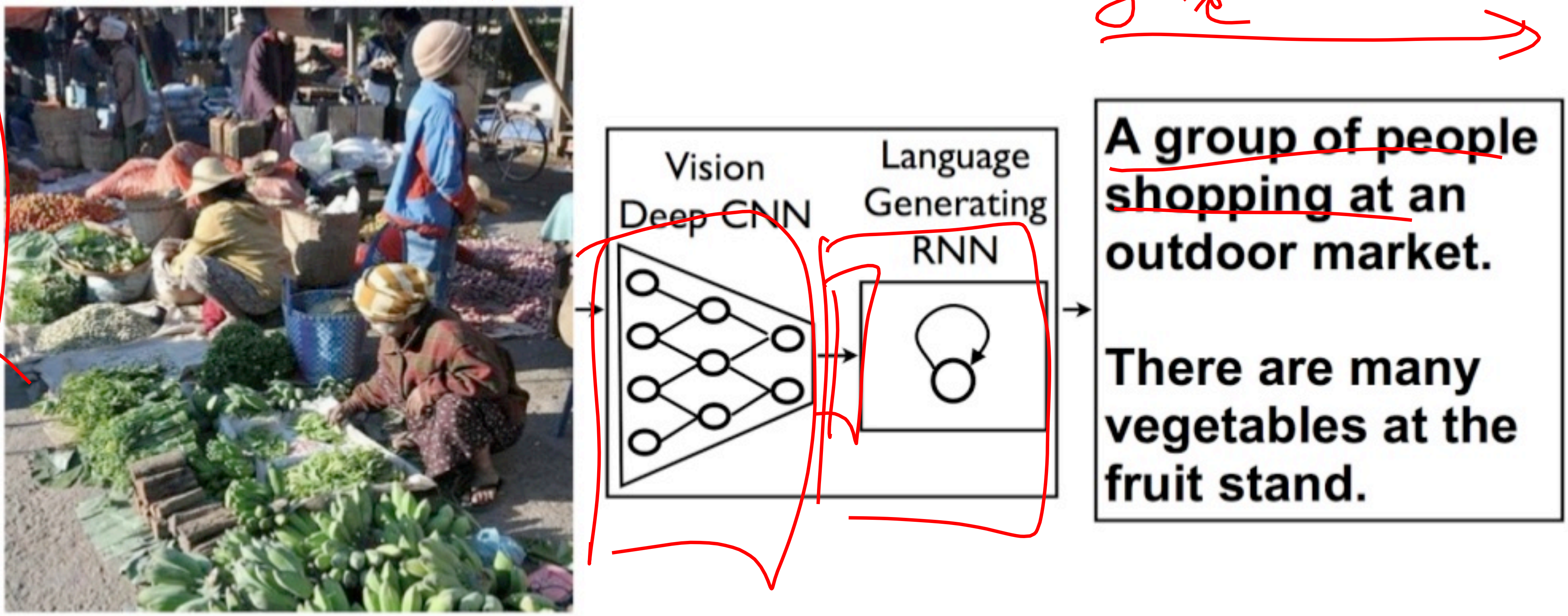
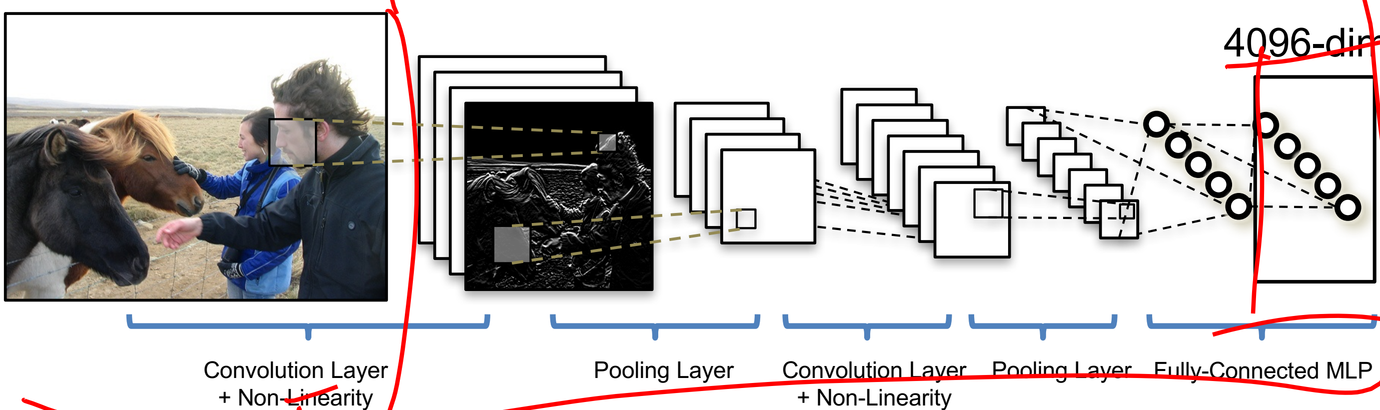


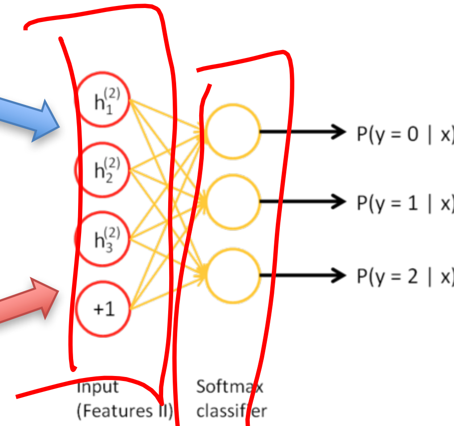
Image Credit: [Vinyals et al. CVPR15]

Visual Question Answering

Image Embedding (VGGNet)



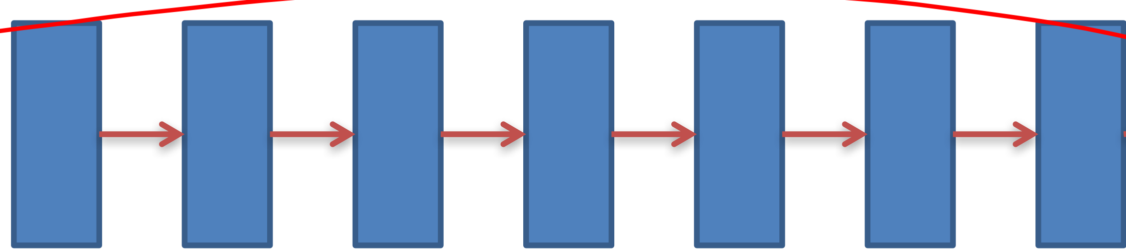
Neural Network
Softmax
over top K answers



$x \in V$

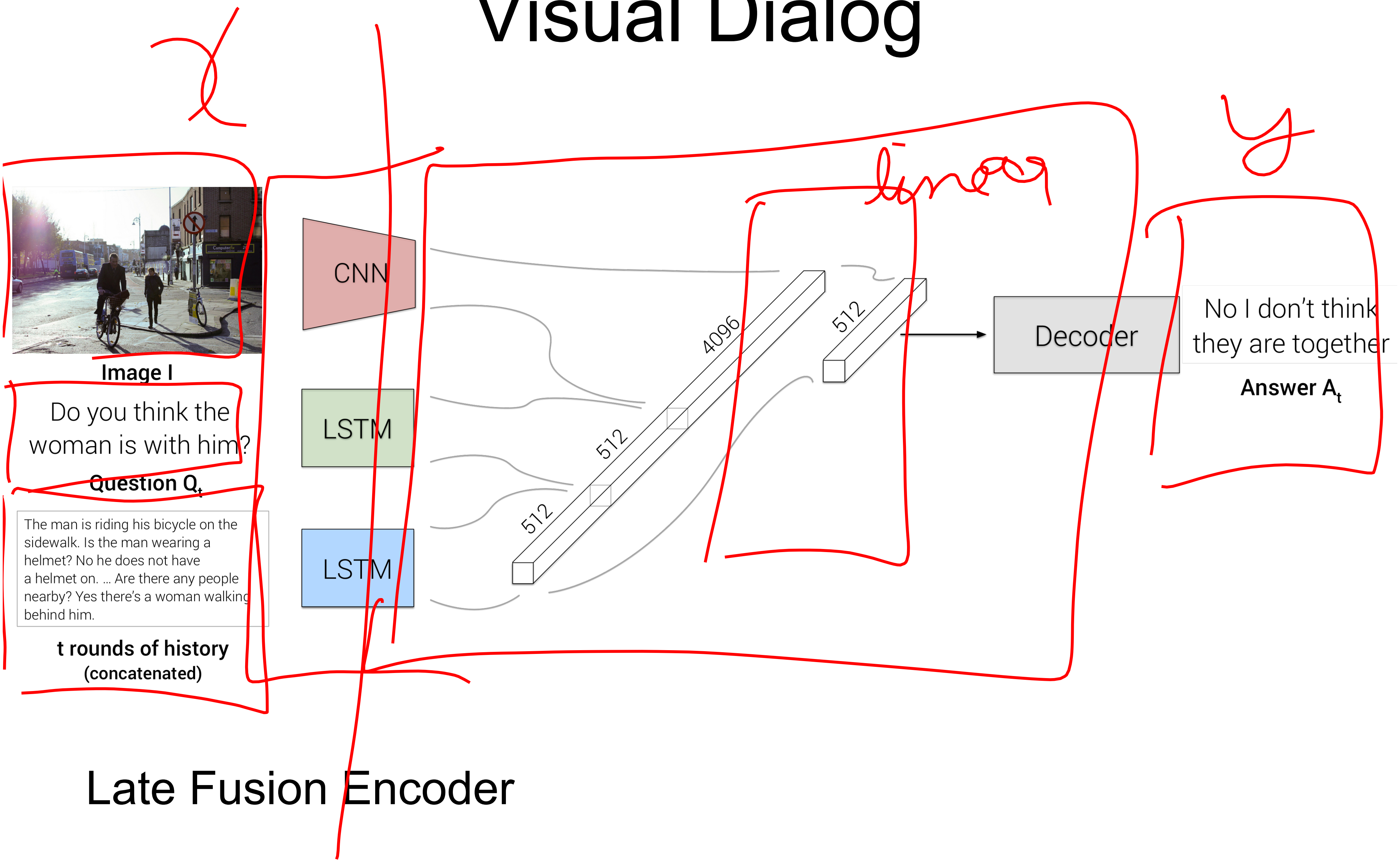
Question Embedding (LSTM)

"How many horses are in this image?"



$y \in \{1 \ldots K\}$

Visual Dialog



Late Fusion Encoder

Slide Credit: Abhishek Das

Recall CIFAR10

50,000 training images
each image is **32x32x3**

10,000 test images.

Parametric Approach

$\vec{x}$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$f(\mathbf{x}, \mathbf{W})$

W

parameters
or weights

Sy

10

10 numbers giving
class scores

Parametric Approach: Linear Classifier

$$f(x,W) = Wx$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

$\longrightarrow f(\mathbf{x},\mathbf{W}) \longrightarrow$ **10** numbers giving class scores

$\uparrow$

W
parameters
or weights

Parametric Approach: Linear Classifier

3072x1

$$f(x,W) = Wx$$

10x1 10x3072

Image

Array of **32x32x3** numbers (3072 numbers total)

$\rightarrow f(\mathbf{x},\mathbf{W}) \rightarrow$ **10** numbers giving class scores

W

parameters or weights

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Parametric Approach: Linear Classifier

$$\underbrace{f(x,W)}_{10\times 1} = \underbrace{W}_{10\times 3072}\overbrace{x}^{3072\times 1} + \underbrace{b}_{10\times 1}$$

Image

Array of **32x32x3** numbers
(3072 numbers total)

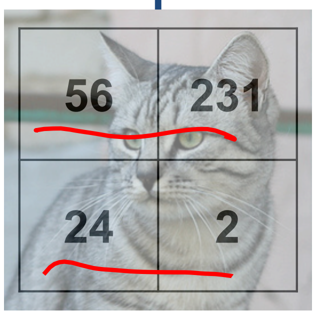
f(**x**,**W**) → **10** numbers giving class scores

W
parameters
or weights

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

$\langle w_{cat}, x \rangle + b_{cat} =$

Stretch pixels into column



Input image

$$
\underbrace{\begin{bmatrix} 0.2 & -0.5 & 0.1 & 2.0 \\ 1.5 & 1.3 & 2.1 & 0.0 \\ 0 & 0.25 & 0.2 & -0.3 \end{bmatrix}}_{W}
\begin{bmatrix} 56 \\ 231 \\ 24 \\ 2 \end{bmatrix}
+
\begin{bmatrix} 1.1 \\ 3.2 \\ -1.2 \end{bmatrix}
=
\begin{array}{c l}
\left[\begin{array}{c} -96.8 \\ 437.9 \\ 61.95 \end{array}\right] &
\begin{array}{l} \text{Cat score} \\ \text{Dog score} \\ \text{Ship score} \end{array}
\end{array}
$$

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

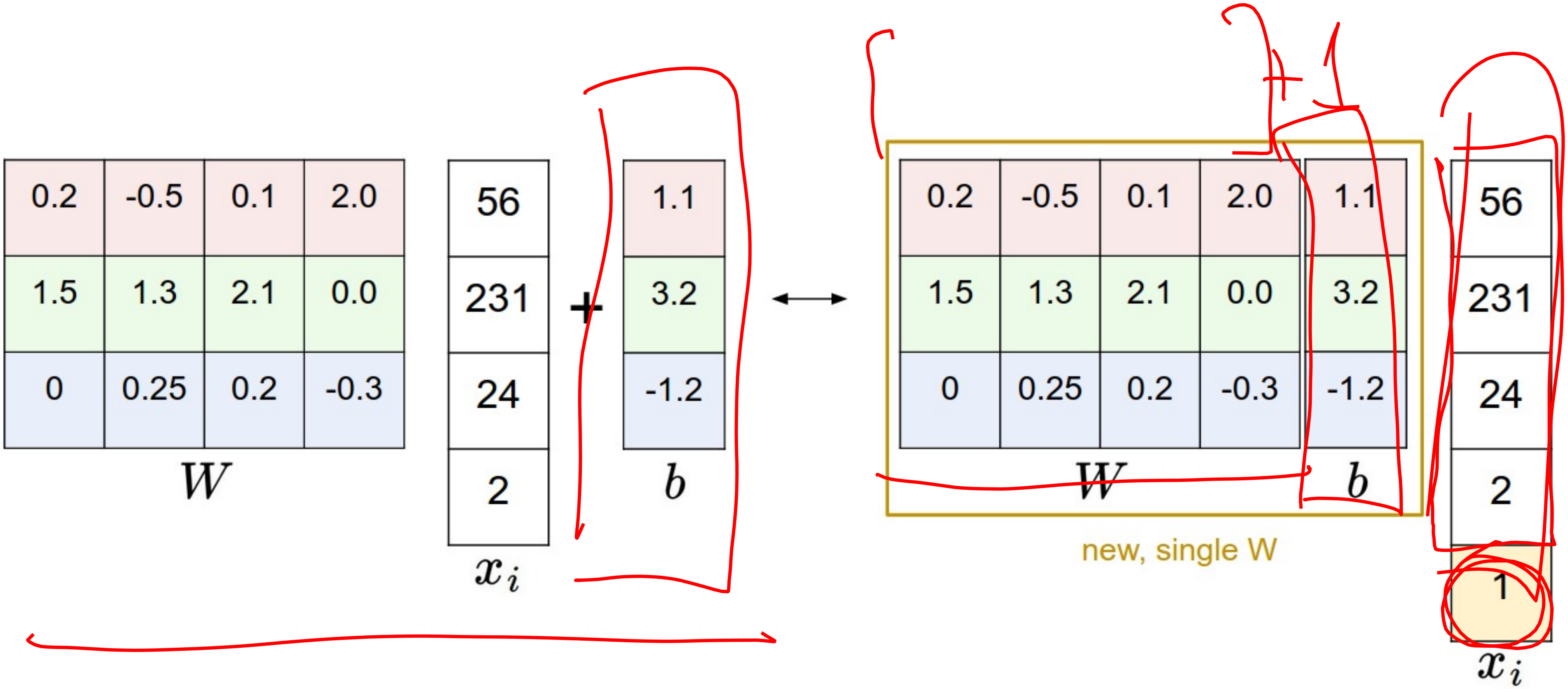


Image Credit: Andrej Karpathy, CS231n

Interpreting a Linear Classifier

$$f(x,W) = Wx + b$$

What is this thing doing?

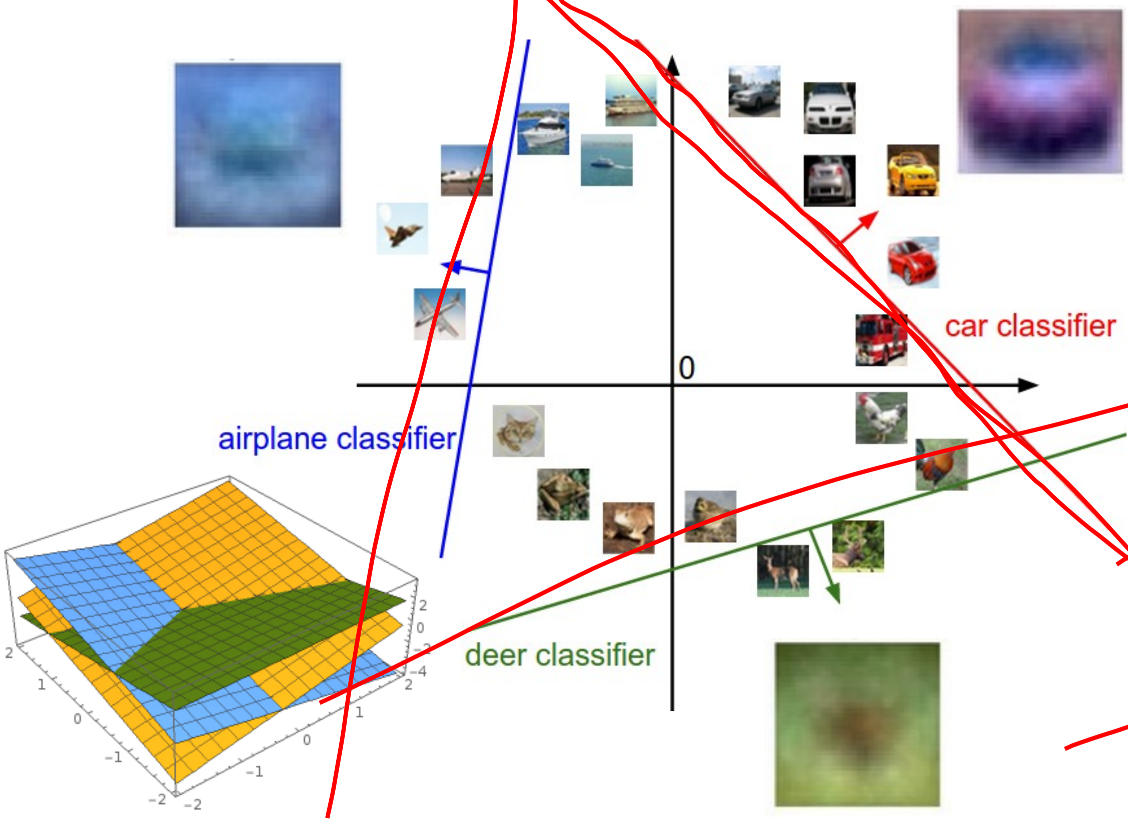
Interpreting a Linear Classifier

$$f(x,W) = Wx + b$$

Example trained weights of a linear classifier trained on CIFAR-10:

Interpreting a Linear Classifier

$\langle w_y, x \rangle + b_y = 0$



$$f(x,W) = \underline{W}x + b$$

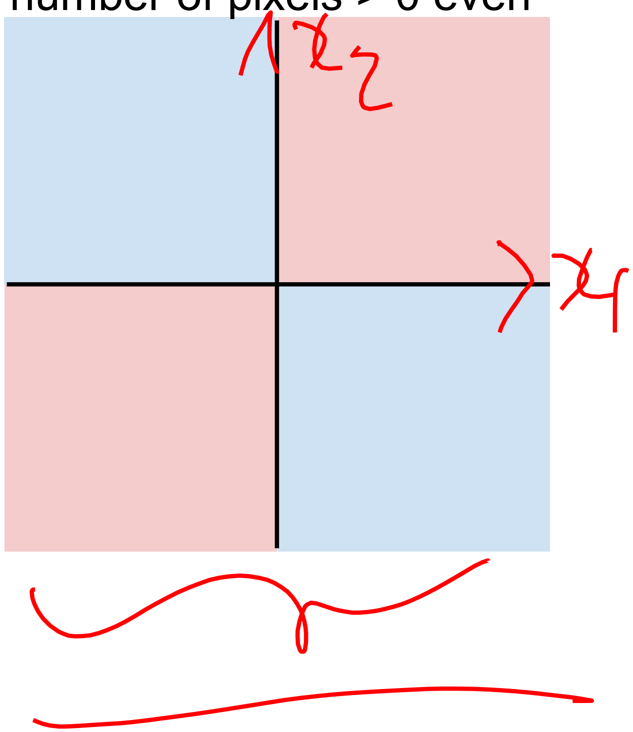
Array of **32x32x3** numbers
(3072 numbers total)

Plot created using Wolfram Cloud

Hard cases for a linear classifier

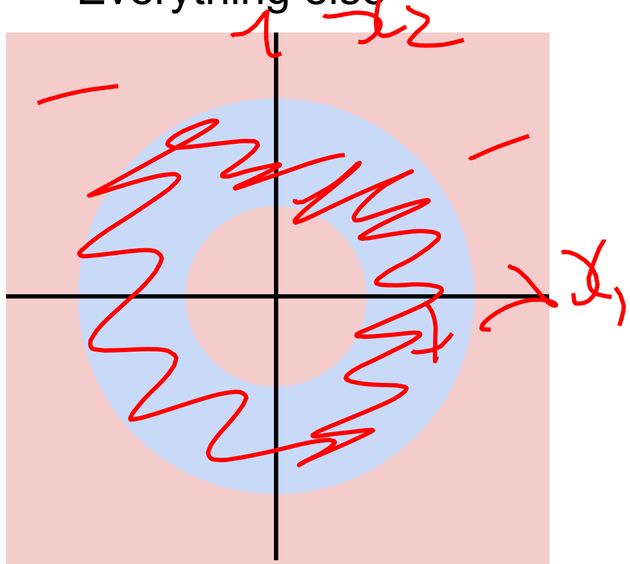
Class 1:
number of pixels > 0 odd

Class 2:
number of pixels > 0 even



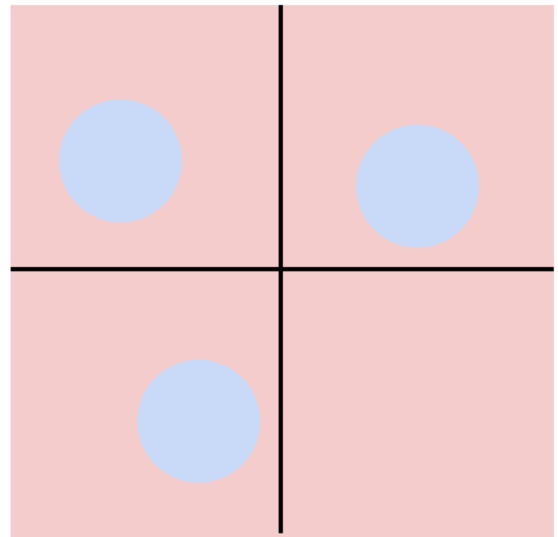
Class 1:
1 <= L2 norm <= 2

Class 2:
Everything else



Class 1:
Three modes

Class 2:
Everything else



So far: Defined a (linear) <u>score function</u>

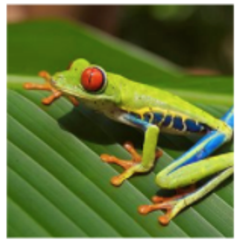
$$f(x,W) = Wx + b$$

airplane	-3.45	-0.51	3.42
automobile	-8.87	**6.04**	4.64
bird	0.09	5.31	2.65
cat	**2.9**	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	**-4.34**
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

Example class scores for 3 images for some W:

How can we tell whether this W is good or bad?

So far: Defined a (linear) score function

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TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.
2. Come up with a way of efficiently finding the parameters that minimize the loss function. **(optimization)**

$\{W, b\}$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**

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A **loss function** tells how good our current classifier is

Given a dataset of examples

$$\{(x_i, y_i)\}_{i=1}^{N}$$

Where x_i is image and y_i is (integer) label

Loss over the dataset is a sum of loss over examples:

$$L = \boxed{\frac{1}{N}} \sum_i \underline{L_i(f(x_i, W), y_i)}$$

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cat	**3.2**	1.3	2.2
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Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \begin{cases} 0 & \text{if } s_{y_i} \geq s_j + 1 \\ s_j - s_{y_i} + 1 & \text{otherwise} \end{cases}$$

$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Suppose: 3 training examples, 3 classes.
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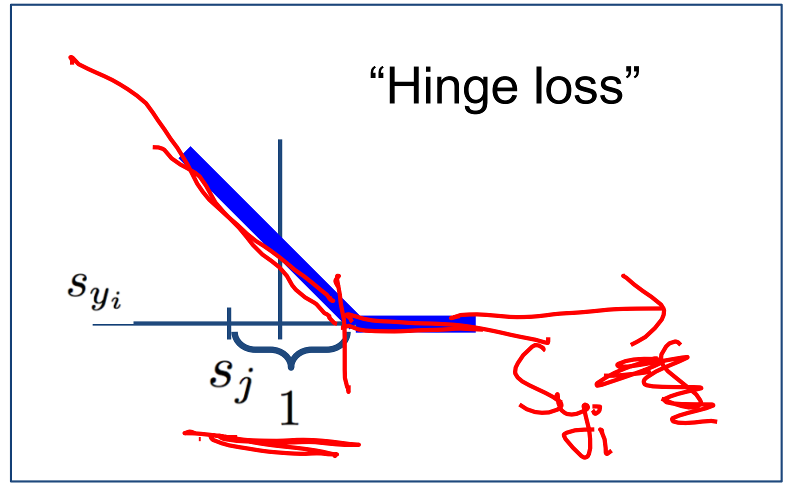

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$$\boxed{L_i = \boxed{\sum_{j \neq y_i}} \max(0, s_j - s_{y_i} + 1)}$$

$$\boxed{\begin{array}{c}\max \\ j \neq y_i\end{array}}$$

Crammer Singer

Suspose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9		

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
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and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 5.1 - 3.2 + 1)
+max(0, -1.7 - 3.2 + 1)
= max(0, 2.9) + max(0, -3.9)
= 2.9 + 0
= 2.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

cat	**3.2**	1.3	2.2
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Losses:	2.9	0	

Multiclass SVM loss:

Given an example (x_i, y_i)
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scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i) where x_i is the image and where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

= max(0, 2.2 - (-3.1) + 1)
+max(0, 2.5 - (-3.1) + 1)
= max(0, 6.3) + max(0, 6.6)
= 6.3 + 6.6
= 12.9

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	**3.2**	1.3	2.2
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the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Loss over full dataset is average:

$$\boxed{L = \frac{1}{N} \sum_{i=1}^{N} L_i}$$

L = (2.9 + 0 + 12.9)/3
= **5.27**

Suppose: 3 training examples, 3 classes.
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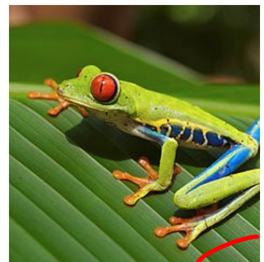

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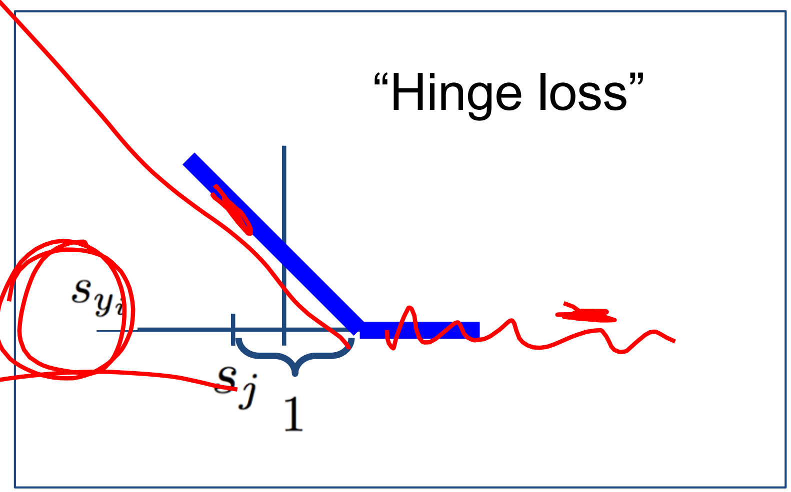
Multiclass SVM loss:



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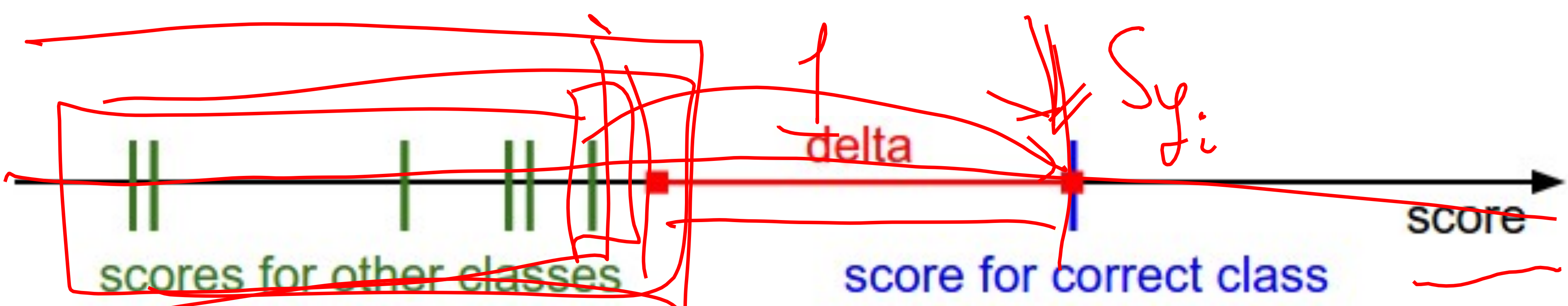
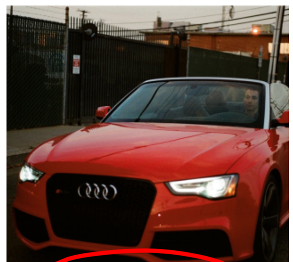
$$= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$



Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



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frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Q: What happens to loss if car image scores change a bit?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
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scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Q2: what is the min/max possible loss?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

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car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: At initialization W is small so all s ≈ 0.
What is the loss?

#classes -1

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

  

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car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

$\frac{1}{N}$

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

2

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Q4: What if the sum
was over all classes?
(including j = y_i)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

			
cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

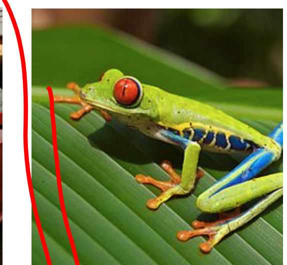
and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Q5: What if we used mean instead of sum?

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	12.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Q6: What if we used

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Multiclass SVM Loss: Example code

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

```
def L_i_vectorized(x, y, W):
  scores = W.dot(x)
  margins = np.maximum(0, scores - scores[y] + 1)
  margins[y] = 0
  loss_i = np.sum(margins)
  return loss_i
```

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^{N} \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

E.g. Suppose that we found a W such that L = 0.
Is this W unique?

No! 2W is also has L = 0!

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:

cat	**3.2**	1.3	2.2
car	5.1	**4.9**	2.5
frog	-1.7	2.0	**-3.1**
Losses:	2.9	0	

$$\boxed{L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)}$$

Before:

= max(0, 1.3 - 4.9 + 1)
+max(0, 2.0 - 4.9 + 1)
= max(0, -2.6) + max(0, -1.9)
= 0 + 0
= 0

With W twice as large:

= max(0, 2.6 - 9.8 + 1)
+max(0, 4.0 - 9.8 + 1)
= max(0, -6.2) + max(0, -4.8)
= 0 + 0
= 0

Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$$\boxed{s = f(x_i; W)}$$

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$$P(Y=k|X=x_i)=\frac{e^{s_k}}{\sum_j e^{s_j}}$$ where $$s=f(x_i;W)$$

cat	**3.2**
car	5.1
frog	-1.7

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$$P(Y = k | X = x_i) = \boxed{\frac{e^{s_k}}{\sum_j e^{s_j}}} \quad \text{where} \quad s = f(x_i; W)$$

Softmax function

cat	**3.2**
car	5.1
frog	-1.7

Why is softmax called softmax?

$$\vec{z} = \begin{bmatrix} z_1 \\ \vdots \\ z_d \end{bmatrix} \xrightarrow{MAX} \begin{bmatrix} 0 \\ \vdots \\ \boxed{z_k} \\ \vdots \\ 0 \end{bmatrix} \qquad k = \underline{\operatorname{argmax}}(z_1, \dots, z_d)$$

$$\begin{bmatrix} z_1 \\ \vdots \\ z_d \end{bmatrix} \longrightarrow \begin{bmatrix} \left(e^{z_i / T} \big/ \sum_{k} e^{z_k / T} \right) \\ \vdots \\ \vdots \end{bmatrix} \qquad T \to 0$$

Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

scores = unnormalized log probabilities of the classes.

$$\boxed{P(Y=k|X=x_i)=\frac{e^{s_k}}{\sum_j e^{s_j}}}$$ where $$\boxed{s=f(x_i;W)}$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$\boxed{L_i=-\log P(Y=y_i|X=x_i)}$$

Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
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scores = unnormalized log probabilities of the classes.

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Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$\boxed{L_i=-\log P(Y=y_i|X=x_i)}$$

in summary: $$L_i=-\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Log-Likelihood / KL-Divergence / Cross-Entropy

$$p^* = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \leftarrow y_i^{gt}$$

$$-\log \hat{p}(y^{gt})$$

$$\hat{p}$$

$$H(p, q)$$

$$P(Y = y \mid \vec{x}_i, w)$$

$$\min KL(p^* \,\|\, \hat{p}) = \sum_y p^*(y) \log \frac{p^*(y)}{\hat{p}(y)}$$

$$= \cancel{-H(p^*)} + H(p^*, \hat{p})$$

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat	**3.2**
car	5.1
frog	-1.7

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat	**3.2**		**24.5**
car	5.1	exp →	164.0
frog	-1.7		0.18

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

	unnormalized log probabilities	$\xrightarrow{\text{exp}}$	unnormalized probabilities	$\xrightarrow{\text{normalize}}$	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

	unnormalized log probabilities	exp →	unnormalized probabilities	normalize →	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
$\boxed{= \mathbf{0.89}}$

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

Q: What is the min/max possible loss L_i?

	unnormalized log probabilities	exp →	unnormalized probabilities	normalize →	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
= **0.89**

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\boxed{\log}\left(\frac{\boxed{e^{s_{y_i}}}}{\sum_j \boxed{e^{s_j}}}\right)$$

unnormalized probabilities

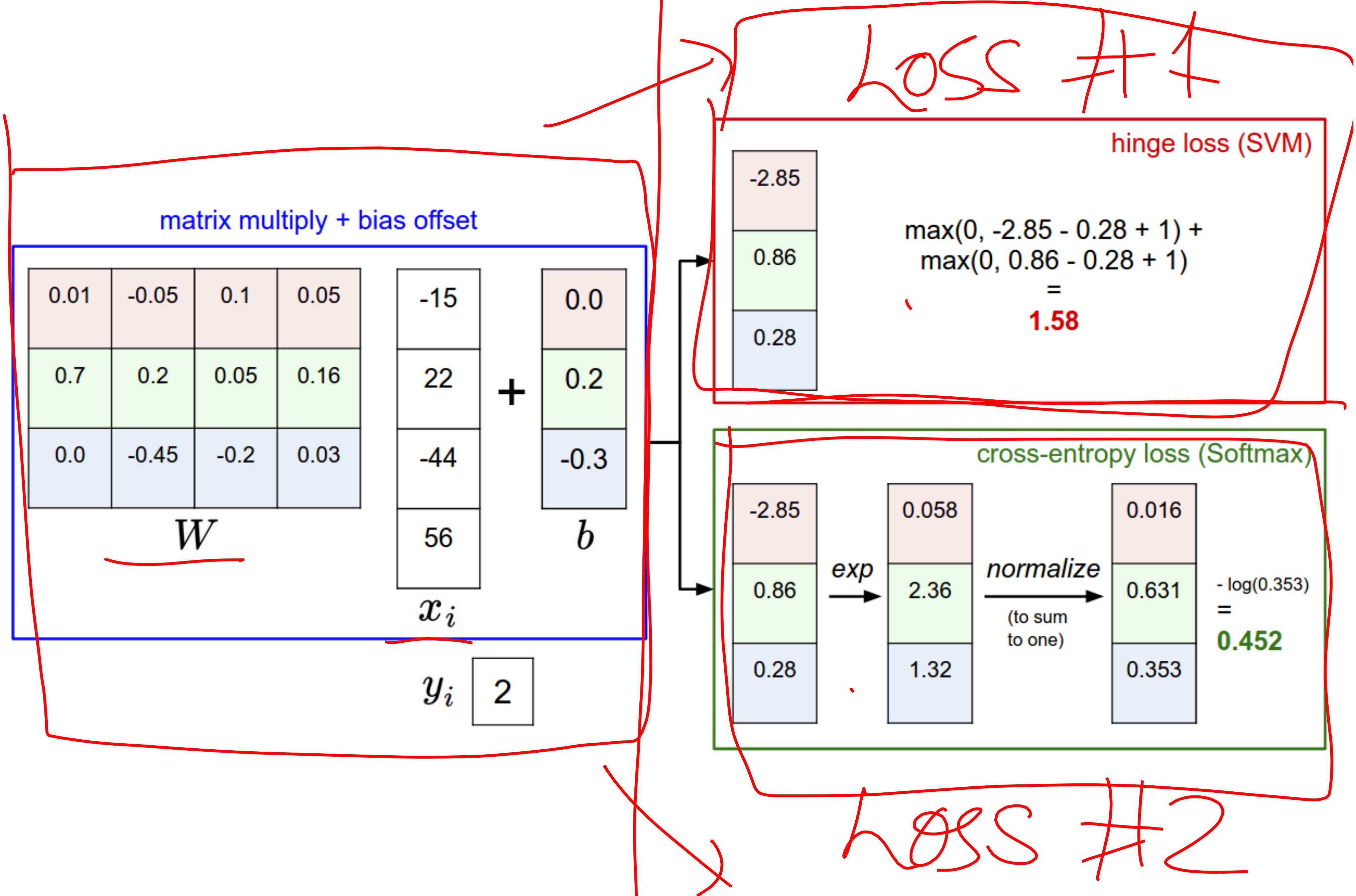
Q2: Usually at initialization W is small so all s ≈ 0.
What is the loss?

	unnormalized log probabilities	exp →	unnormalized probabilities	normalize →	probabilities
cat	**3.2**		**24.5**		**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
= **0.89**

log K

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n


Loss #1
hinge loss (SVM)
-2.85
0.86
0.28
max(0, -2.85 - 0.28 + 1) +
max(0, 0.86 - 0.28 + 1)
=
1.58
matrix multiply + bias offset
0.01
-0.05
0.1
0.05
0.7
0.2
0.05
0.16
0.0
-0.45
-0.2
0.03
W
-15
22
-44
56
x_i
+
0.0
0.2
-0.3
b
y_i
2
cross-entropy loss (Softmax)
-2.85
0.86
0.28
exp
0.058
2.36
1.32
normalize
(to sum
to one)
0.016
0.631
0.353
- log(0.353)
=
0.452
Loss #2

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right) \qquad\qquad L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

[10, 9, 9]

[10, -100, -100]

and $\boxed{y_i = 0}$

Q: Suppose I take a datapoint and I jiggle a bit (changing its score slightly). What happens to the loss in both cases?

Recap

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

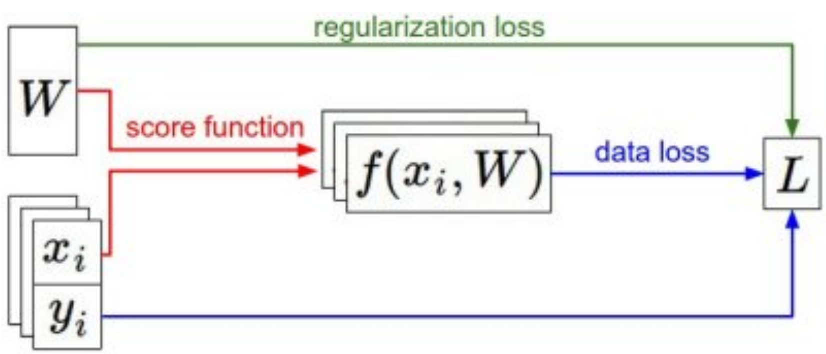
Softmax

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



Recap

How do we find the best W?

- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

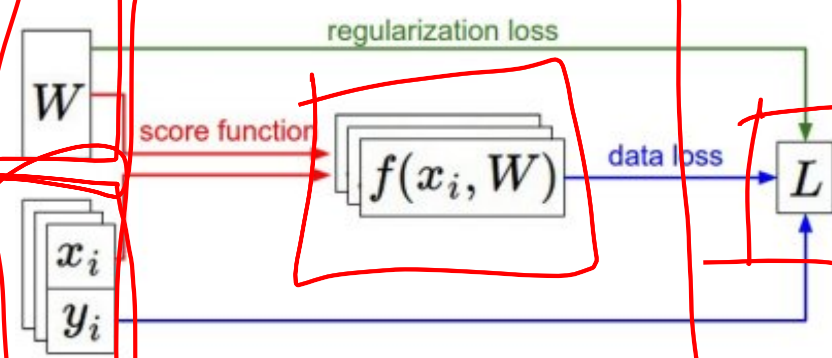
Softmax

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

SVM

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$ Full loss



$$\frac{\partial L}{\partial W}$$

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n