CS 4644-DL / 7643-A: LECTURE 8
DANFEI XU

Topics:

- Convolution and Convolution Layers
- Pooling
- Convolutional Neural Networks Architectures (Part 1)

Convolutional Neural Networks

Recall:

Hubel & Wiesel,

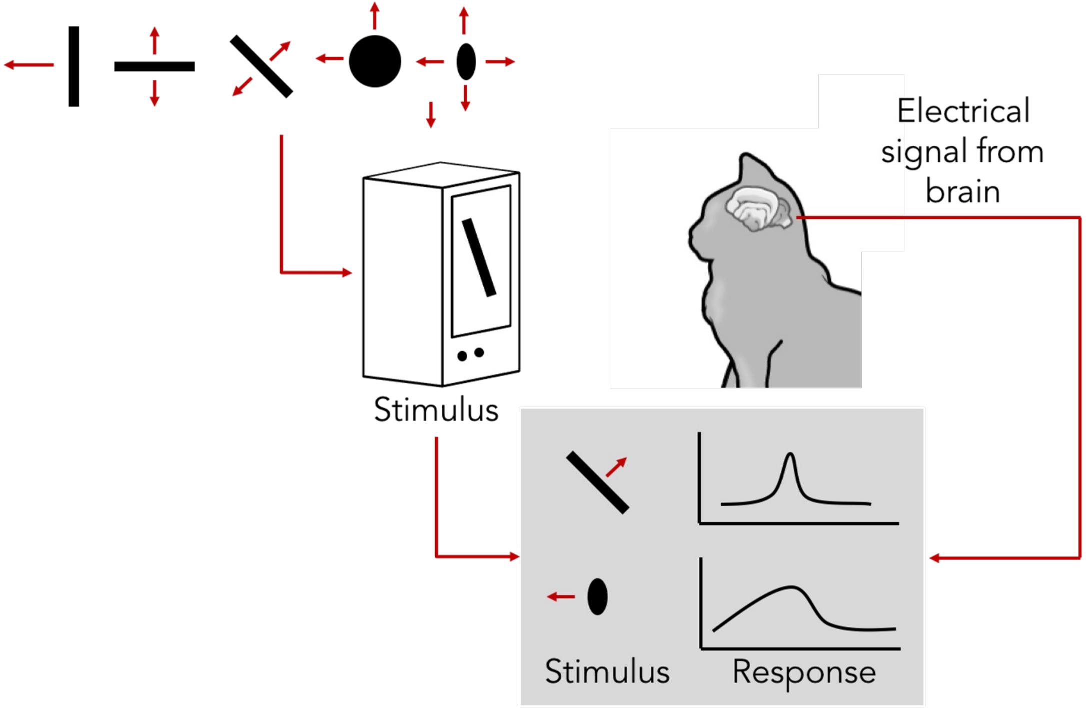
1959

RECEPTIVE FIELDS OF SINGLE NEURONES IN
THE CAT'S STRIATE CORTEX

1962

RECEPTIVE FIELDS, BINOCULAR INTERACTION
AND FUNCTIONAL ARCHITECTURE IN THE CAT'S VISUAL CORTEX

1968...



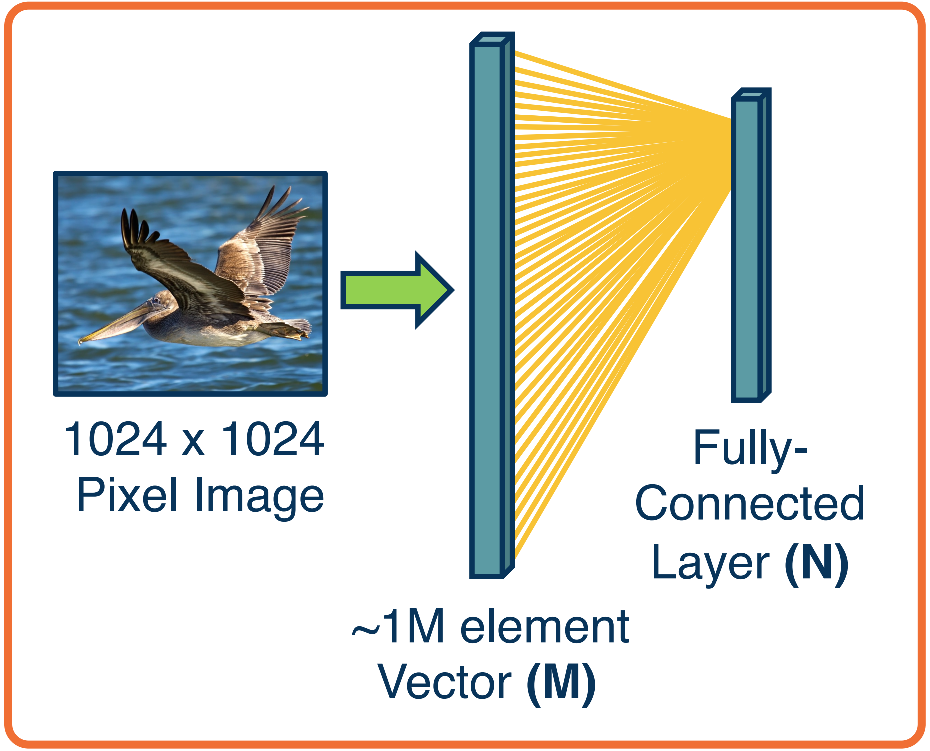
Slide credit: Stanford CS231n Instructors

Image features are spatially localized!

- Relevant features repeated across the image
  - Edges
  - Color
  - Motifs (corners, etc.)
- No reason to believe one feature tends to appear in a fixed location. Need to search in entire image.

Can we enforce a structure in the design of a neural network layer to reflect this?

The connectivity in linear layers **doesn't always make sense**



How many parameters?

- M*N (weights) + N (bias)

Hundreds of millions of parameters **for just one layer**

More parameters => More data needed

Can we design a layer with localized connection?

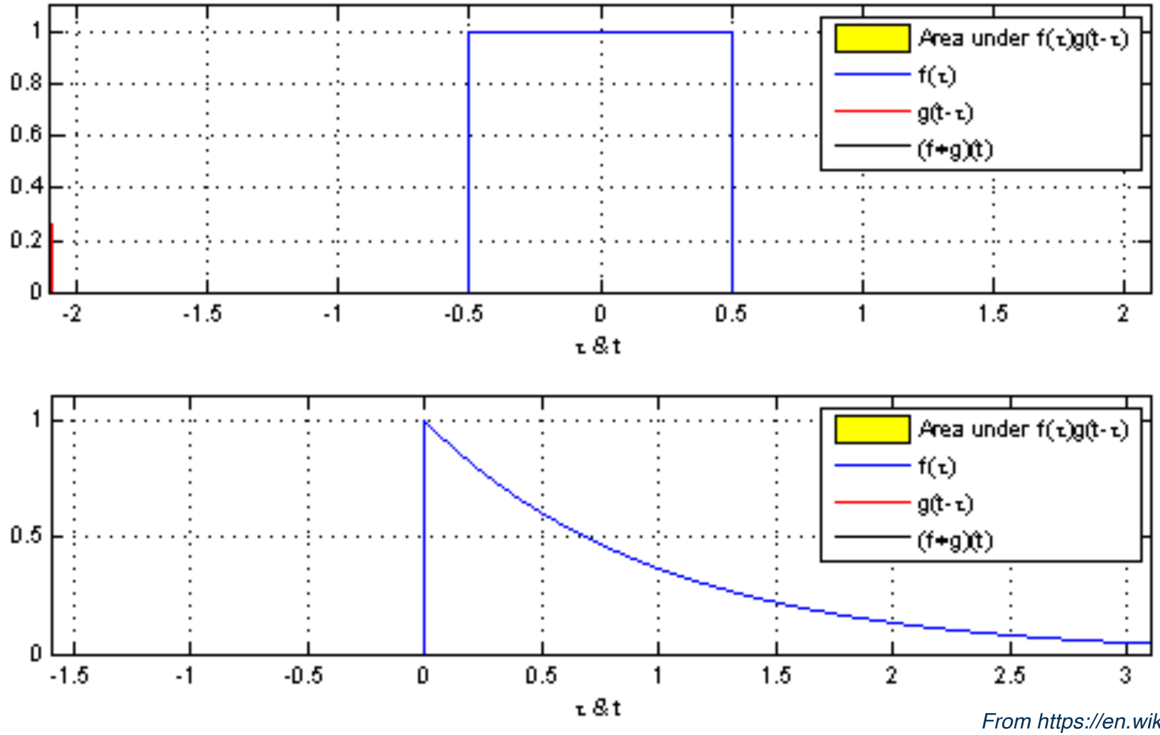
Limitation of Linear Layers

Convolution: A 1D Visual Example

input f

filter g

response $f * g$



From https://en.wikipedia.org/wiki/Convolution

Intuitively, we seek to learn **neural conv filters** that looks for patterns in the input

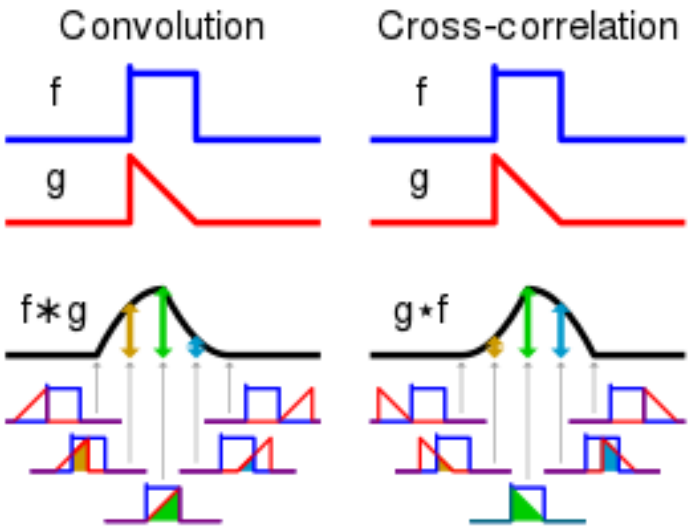
Convolution

1-D Convolution is defined as the **integral** of the **product** of two functions after one is reflected about the y-axis and shifted.

Cross-correlation is convolution without the y-axis reflection.

Intuitively: given function f and filter g. How similar is $g(-x)$ with the part of $f(x)$ that it's operating on.

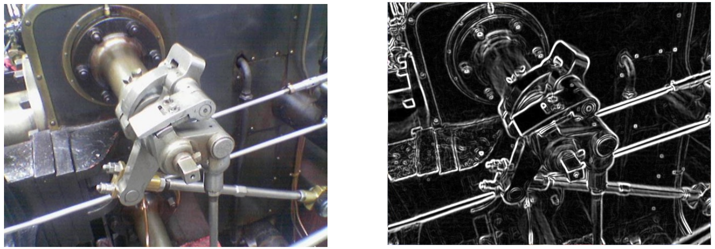
For ConvNets, we don't flip filters, so we are really using Cross-Correlation Nets!



From https://en.wikipedia.org/wiki/Convolution

Convolution in Computer Vision (non-Deep)

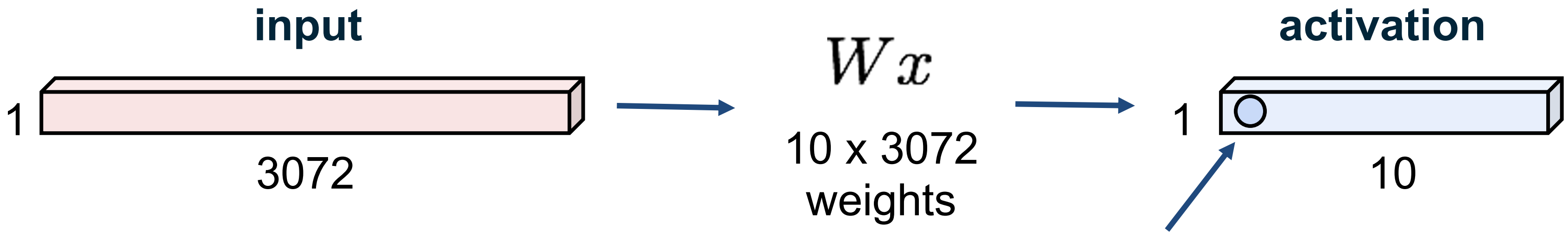
Convolution with Gaussian Filter (Gaussian Blur)



Convolution with Sobel Filter (Edge Detection)

Fully Connected Layer

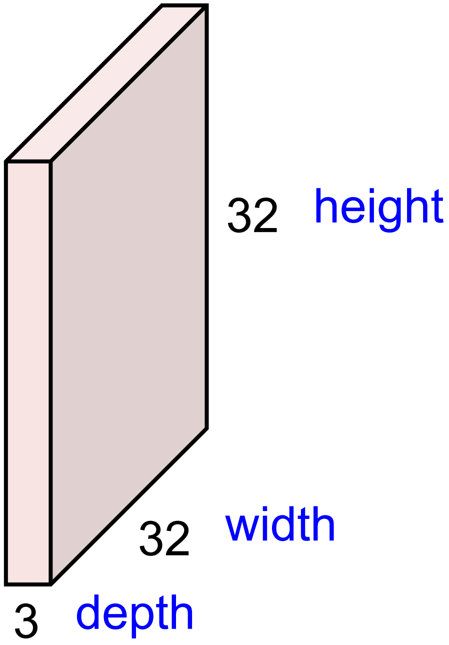
32x32x3 image -> stretch to 3072 x 1



1 number:
the result of taking a dot product between a row of W and the input (a 3072-dimensional dot product)

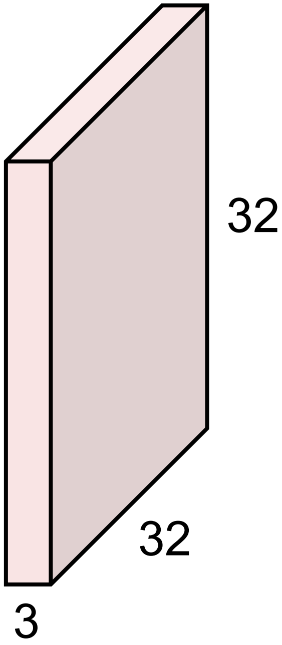
Convolution Layer

32x32x3 image -> preserve spatial structure

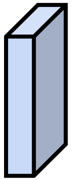


Convolution Layer

32x32x3 image



5x5x3 filter

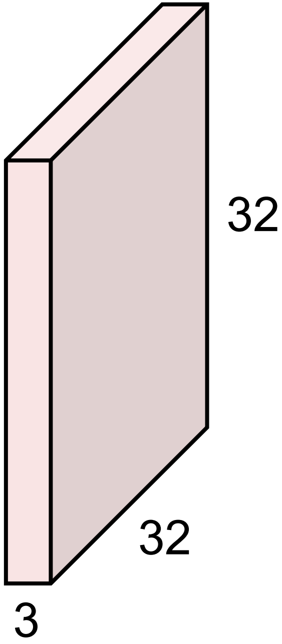


Convolve the filter with the image
i.e. “slide over the image spatially,
computing dot products”

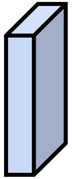
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image



5x5x3 filter

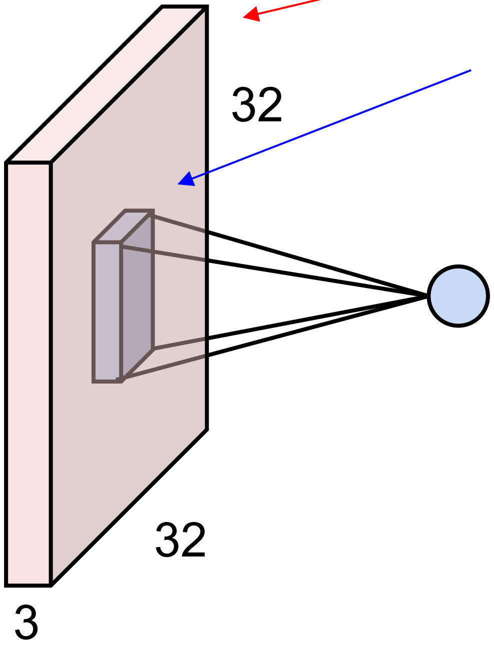


Convolve the filter with the image
i.e. “slide over the image spatially, computing dot products”

Convolution Layer

32x32x3 image

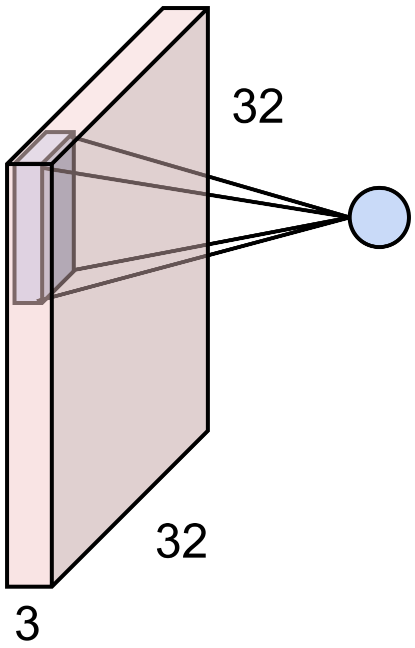
5x5x3 filter w



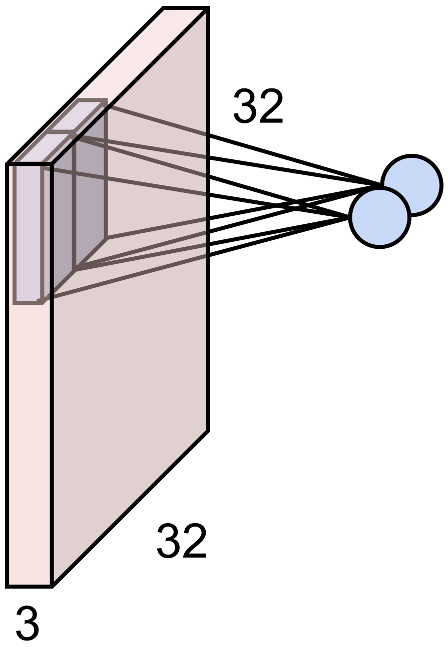
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

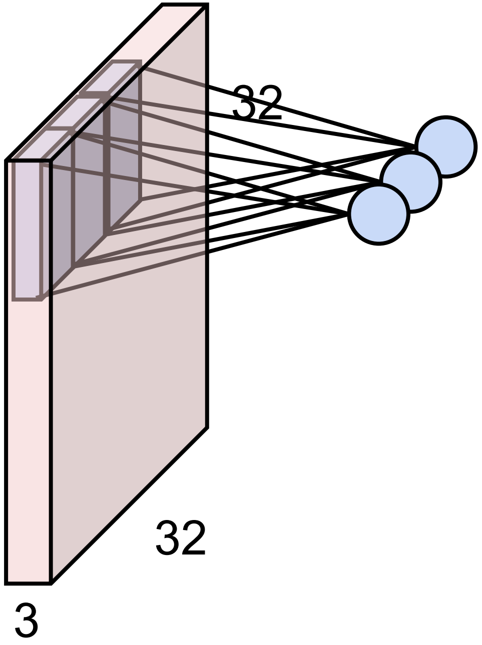
Convolution Layer



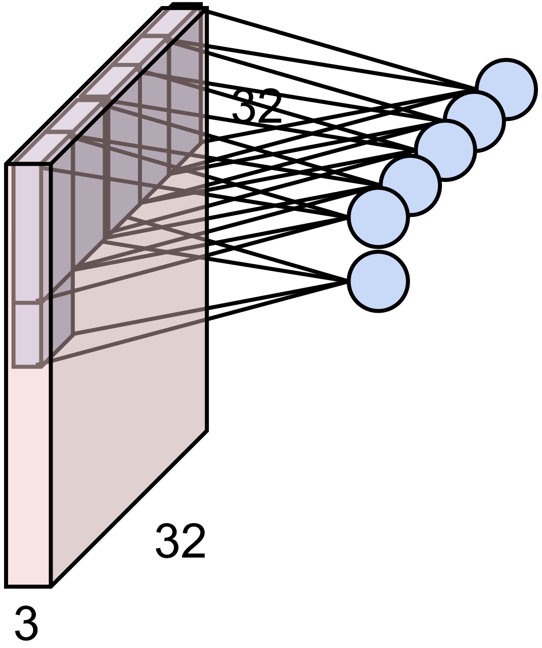
Convolution Layer



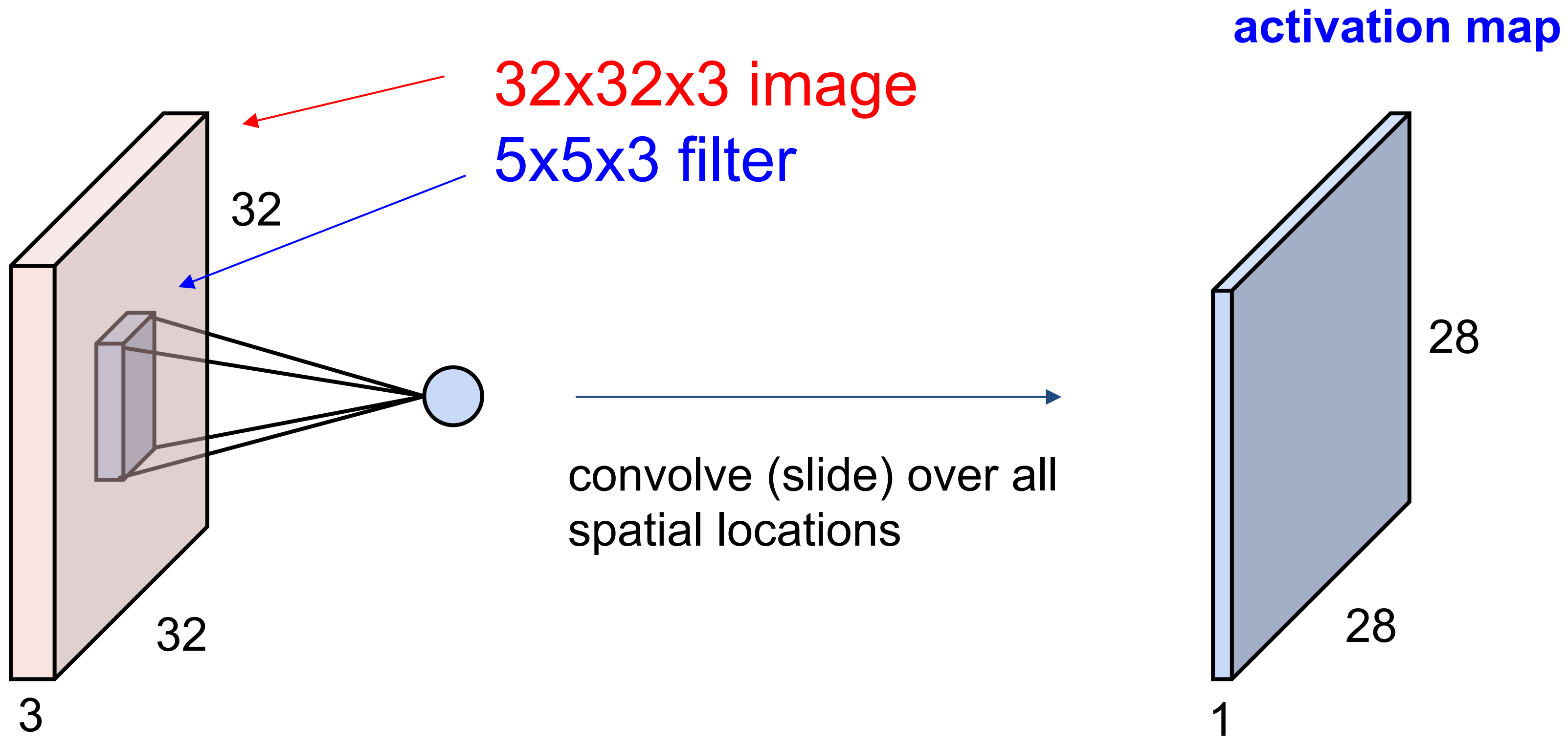
Convolution Layer



Convolution Layer

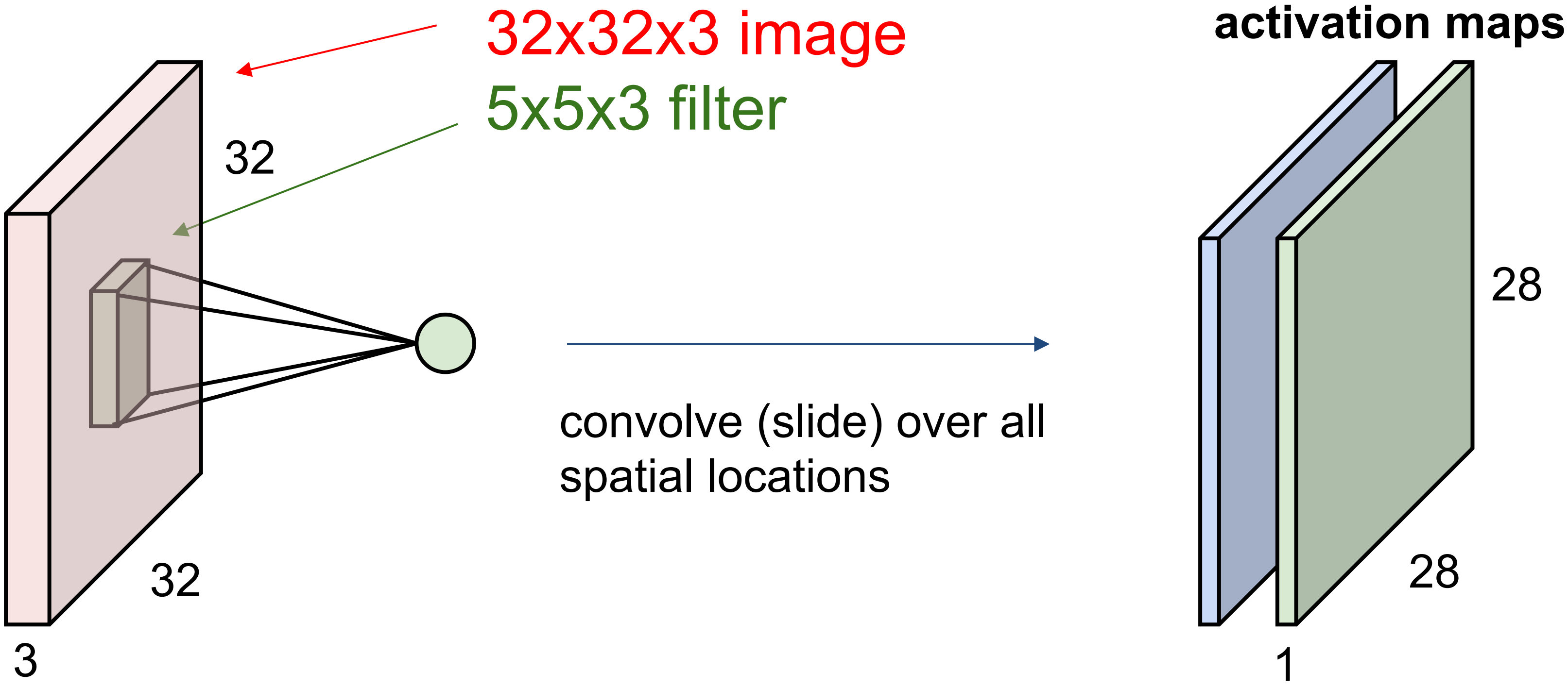


Convolution Layer

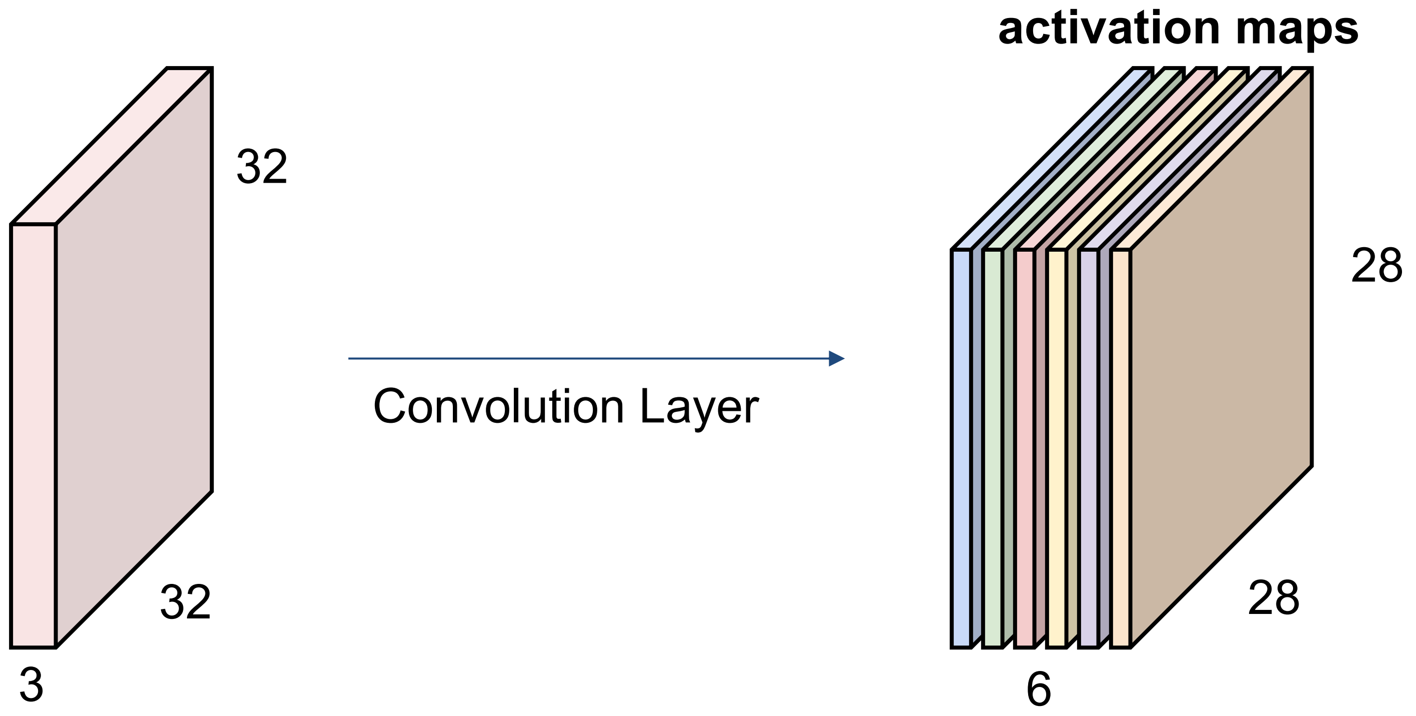


Convolution Layer

consider a second, green filter

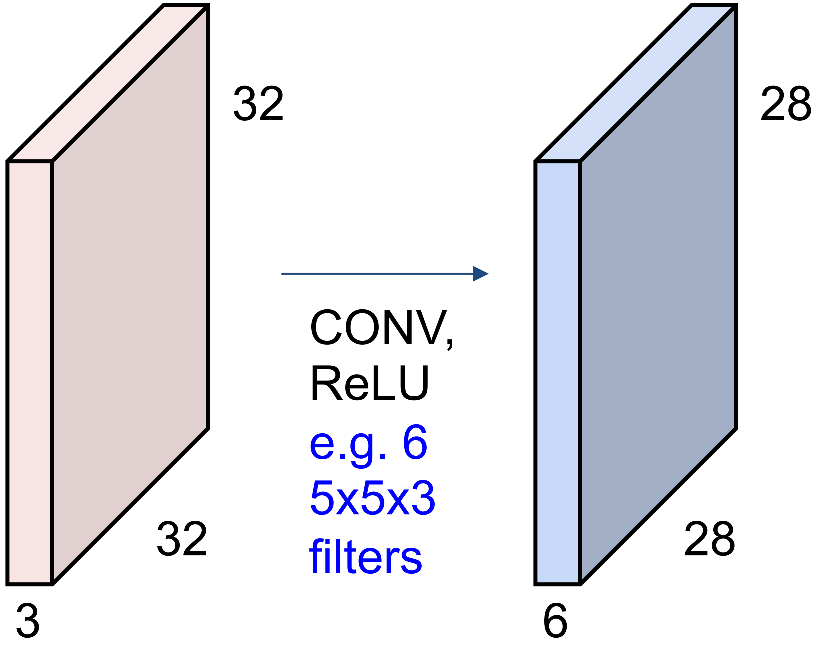


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

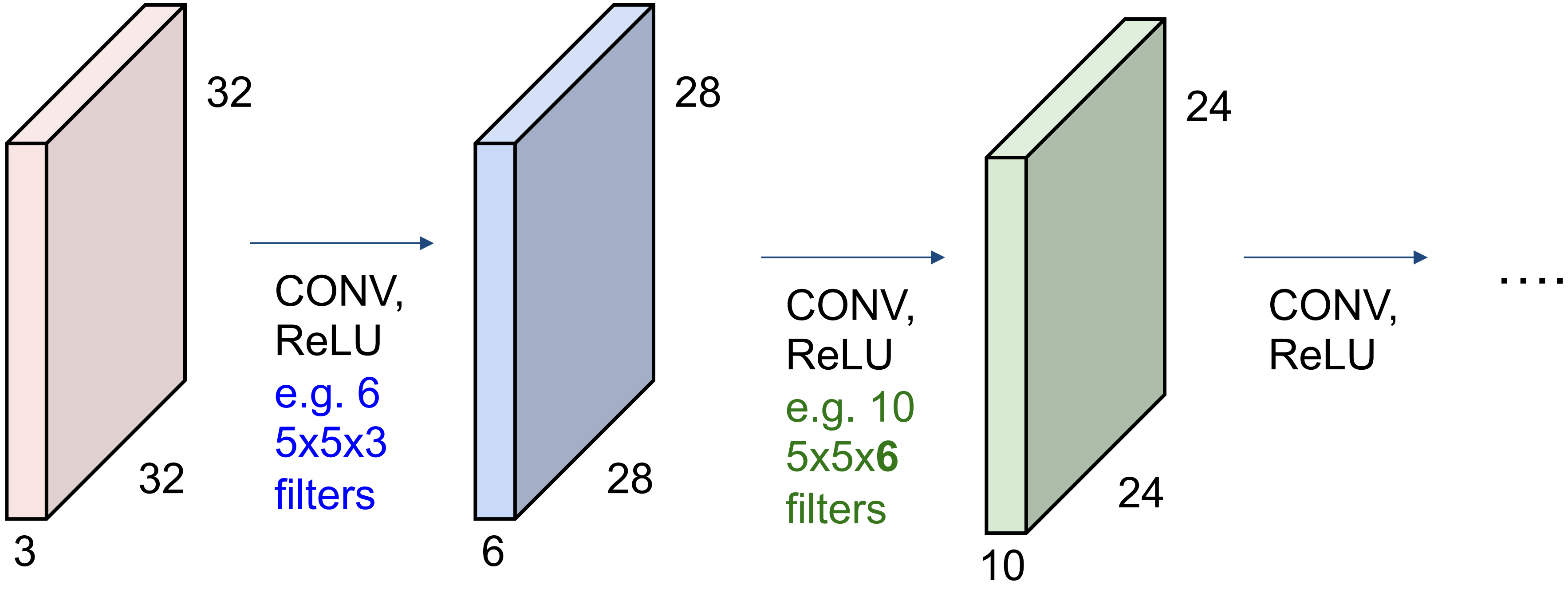


We stack these up to get a "new image" of size 28x28x6!

Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



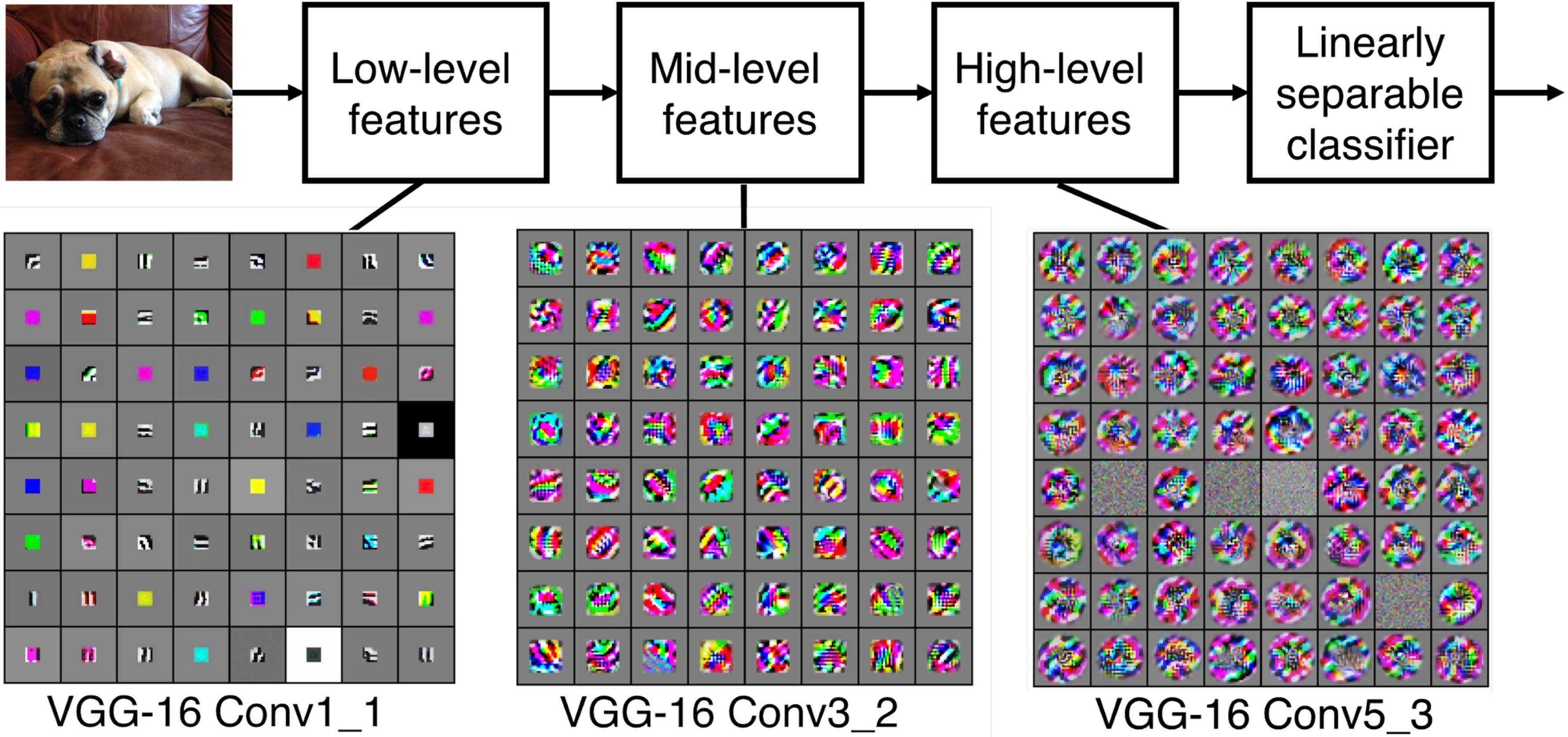
Preview: ConvNet is a sequence of Convolution Layers, interspersed with activation functions



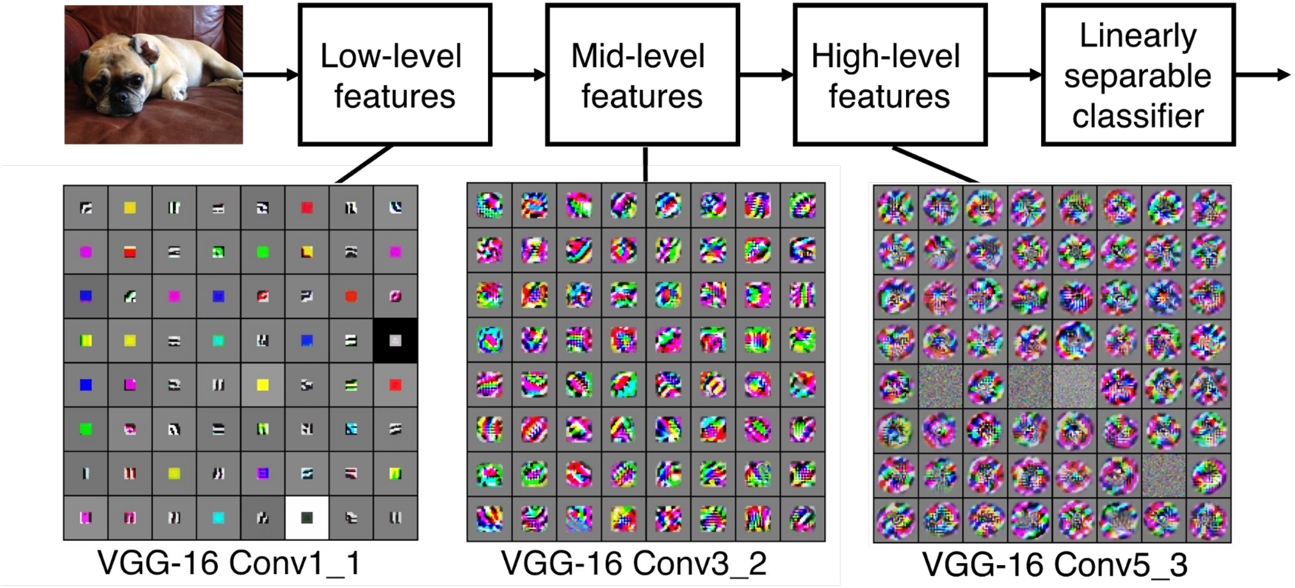
Preview

[Zeiler and Fergus 2013]

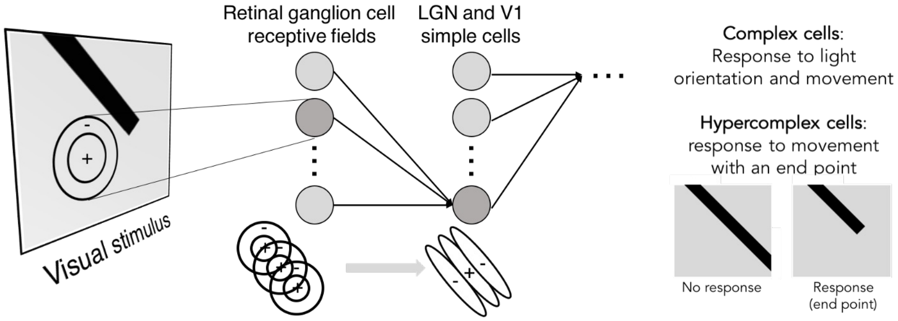
Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].



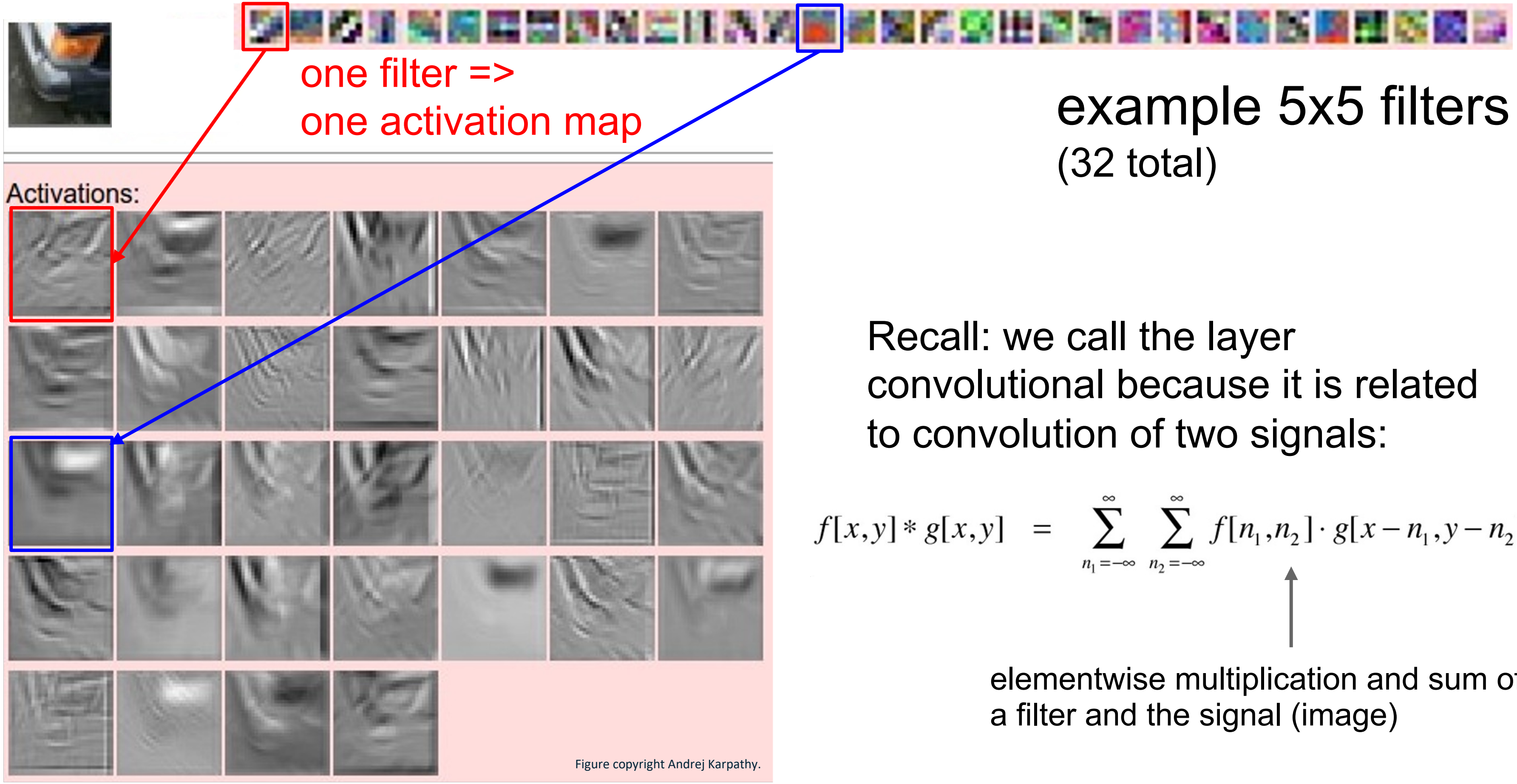
Preview



Low-level features
Mid-level features
High-level features
Linearly separable classifier
VGG-16 Conv1_1
VGG-16 Conv3_2
VGG-16 Conv5_3



Retinal ganglion cell receptive fields
LGN and V1 simple cells
Visual stimulus
Complex cells:
Response to light orientation and movement
Hypercomplex cells:
response to movement with an end point
No response
Response (end point)

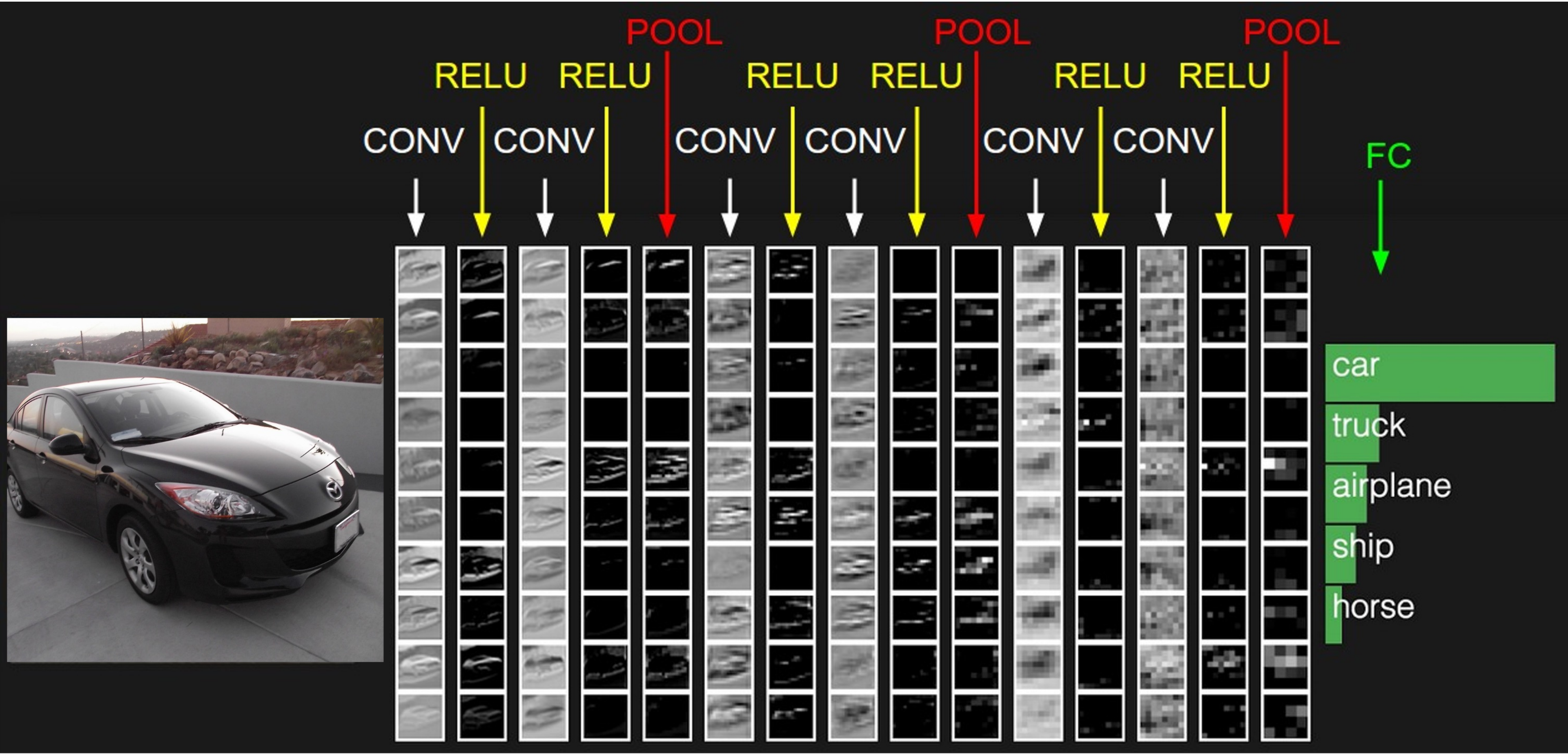


example 5x5 filters
(32 total)

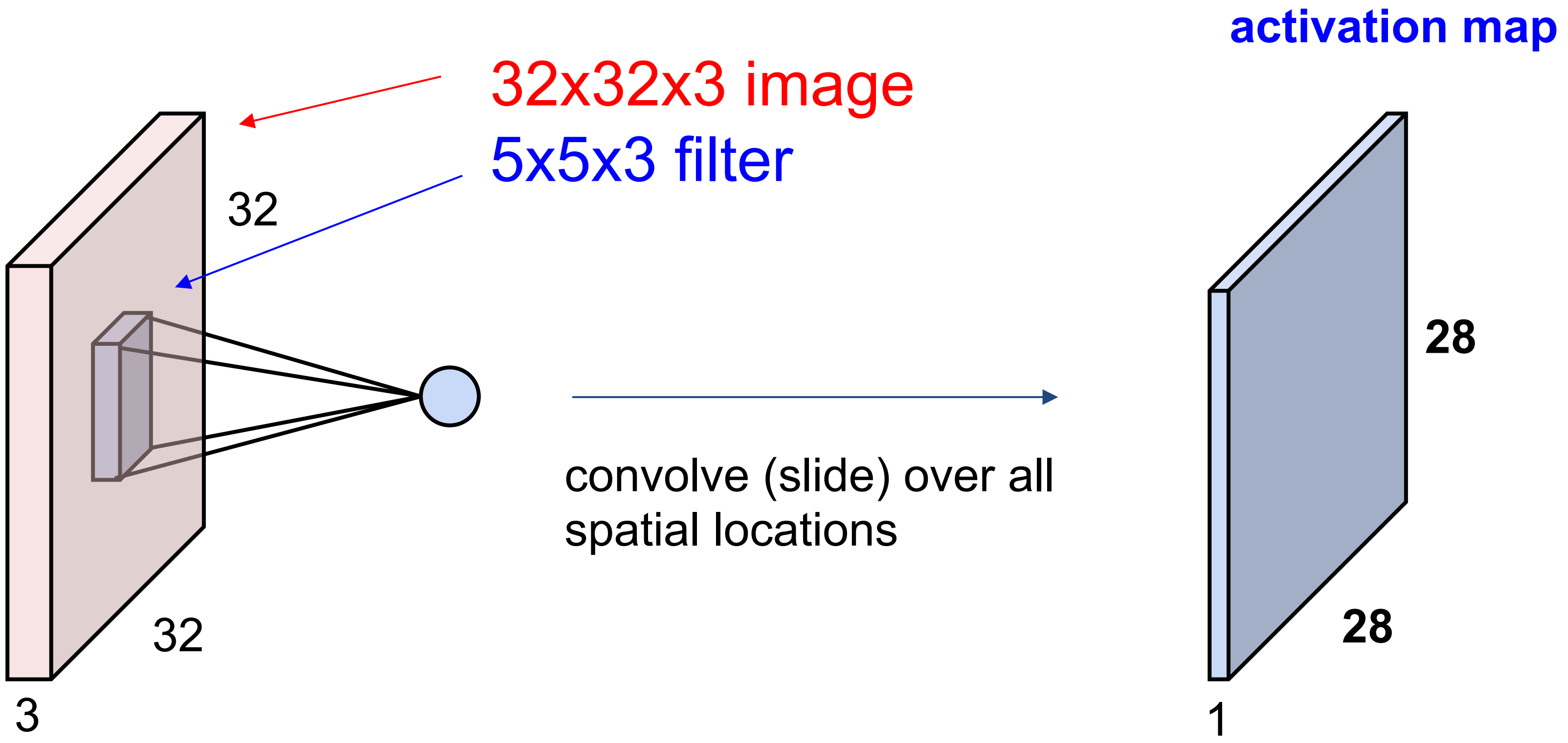
Recall: we call the layer convolutional because it is related to convolution of two signals:

$$f[x,y] * g[x,y] = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f[n_1,n_2] \cdot g[x-n_1, y-n_2]$$

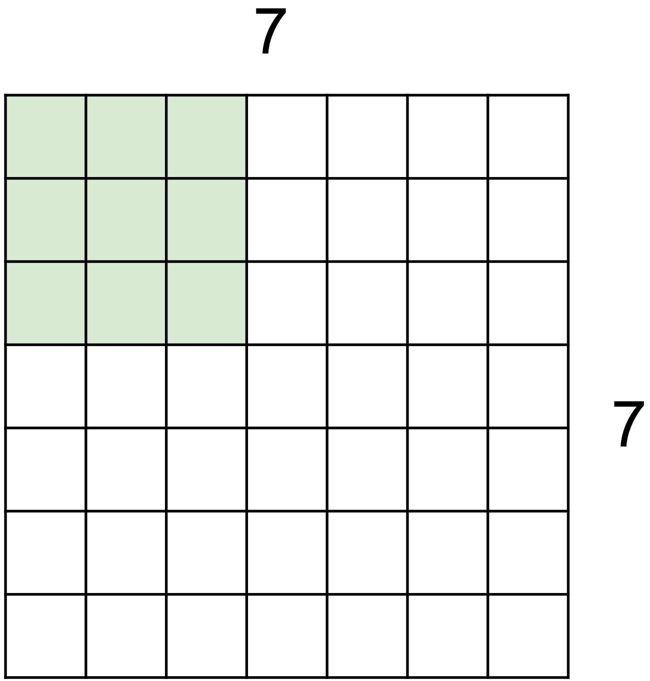
elementwise multiplication and sum of a filter and the signal (image) preview:


POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

A closer look at spatial dimensions:

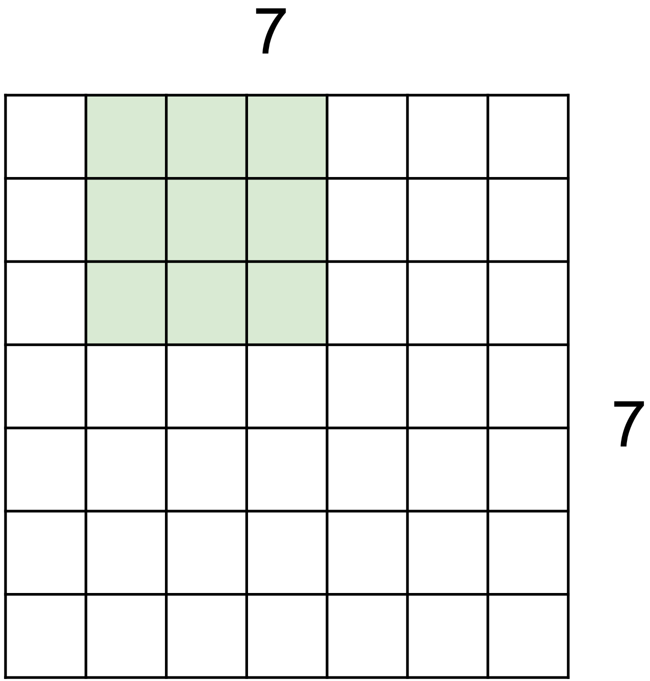


A closer look at spatial dimensions:



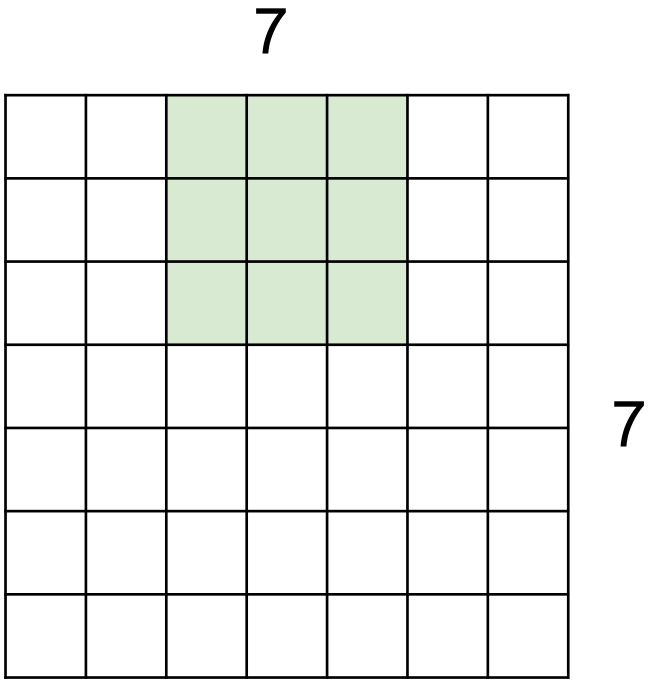
7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter

A closer look at spatial dimensions:

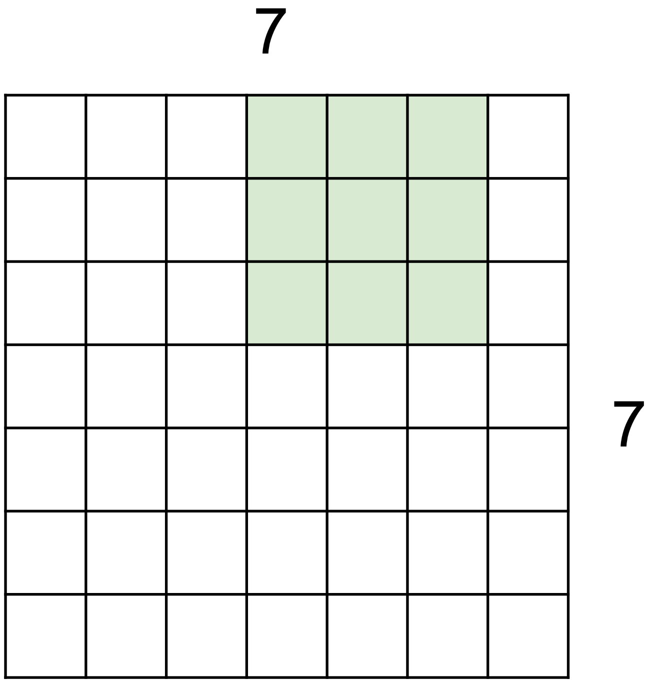


7x7 input (spatially)
assume 3x3 filter

The # of grid that the filter shifts is called **stride.**

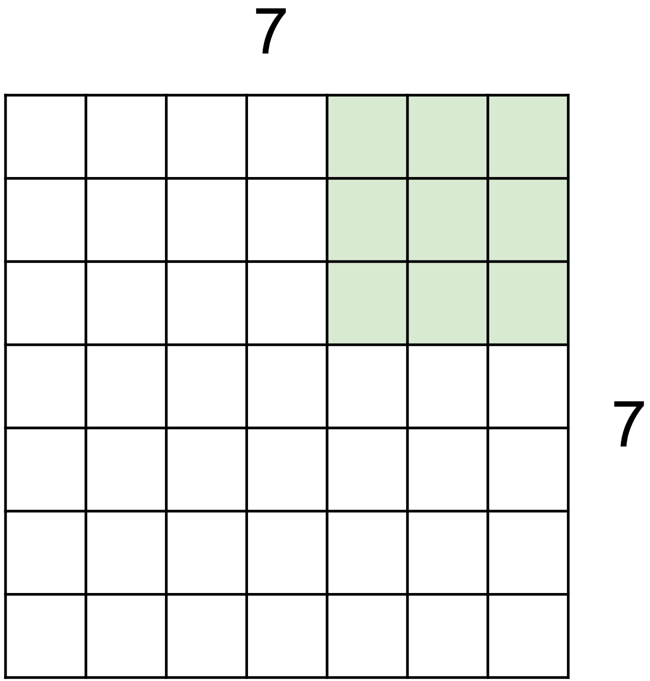
E.g., here we have stride = 1

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter **with stride = 1**

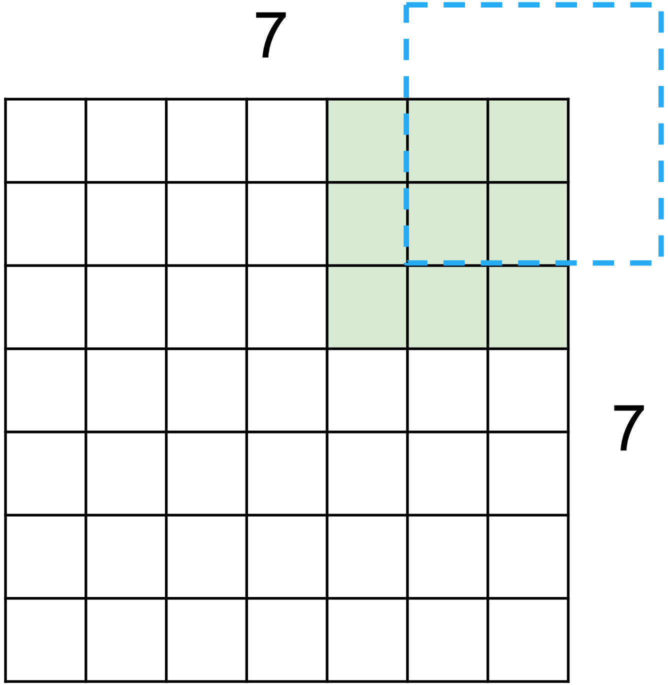
A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter with stride = 1

=> 5x5 output

A closer look at spatial dimensions:

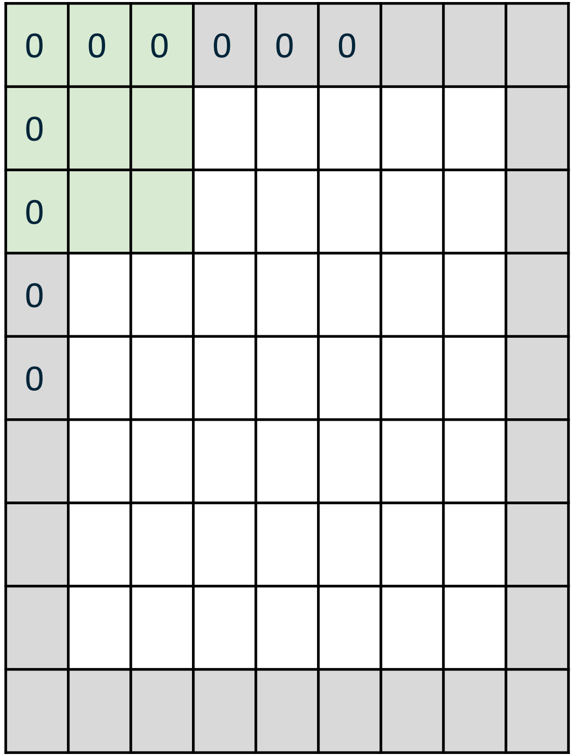


7x7 input (spatially)
assume 3x3 filter with stride = 1

=> 5x5 output

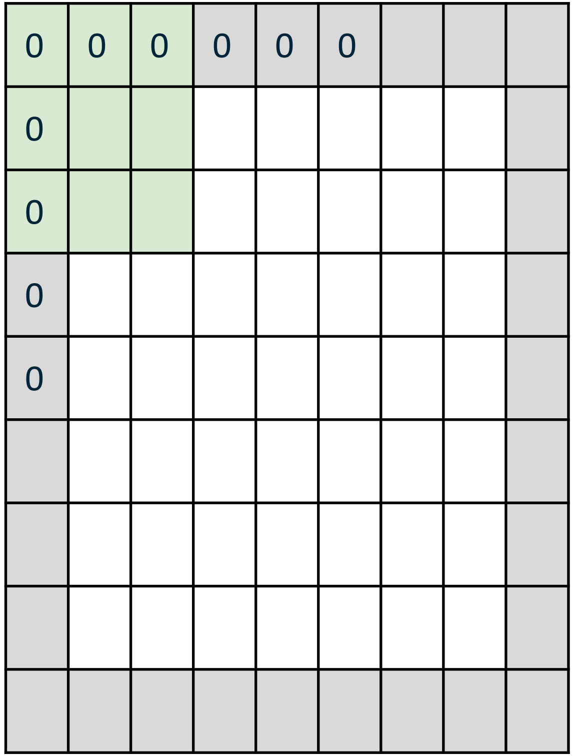
But what about the features at the border?

In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

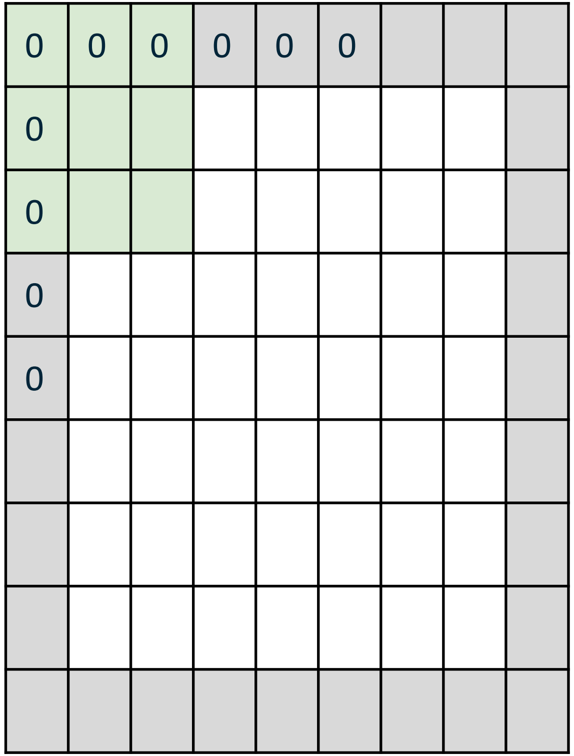
In practice: Common to zero pad the border



e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!
in general, common to see CONV layers with stride 1, filters of size FxF, and zero-padding with (F-1)/2. (will preserve size spatially)
e.g. F = 3 => zero pad with 1
F = 5 => zero pad with 2
F = 7 => zero pad with 3

In practice: Common to zero pad the border

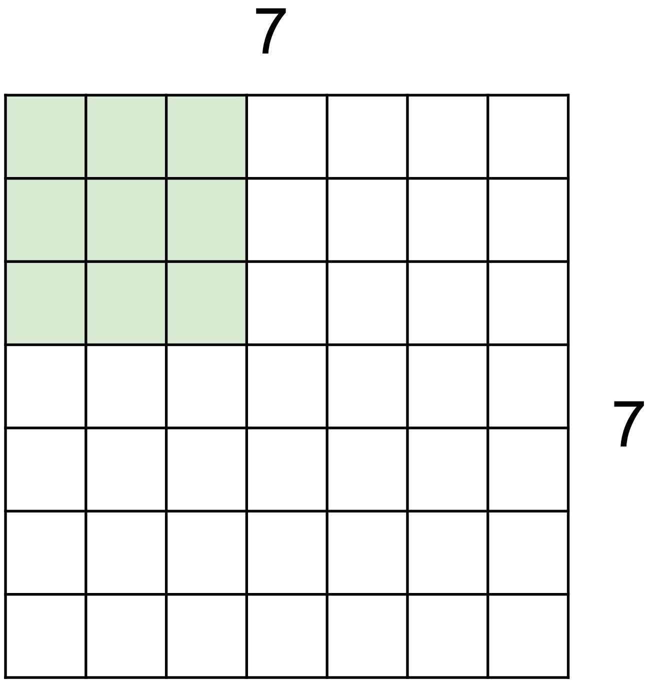


e.g. input 7x7
3x3 filter, applied with **stride 1**
pad with 1 pixel border => what is the output?

7x7 output!

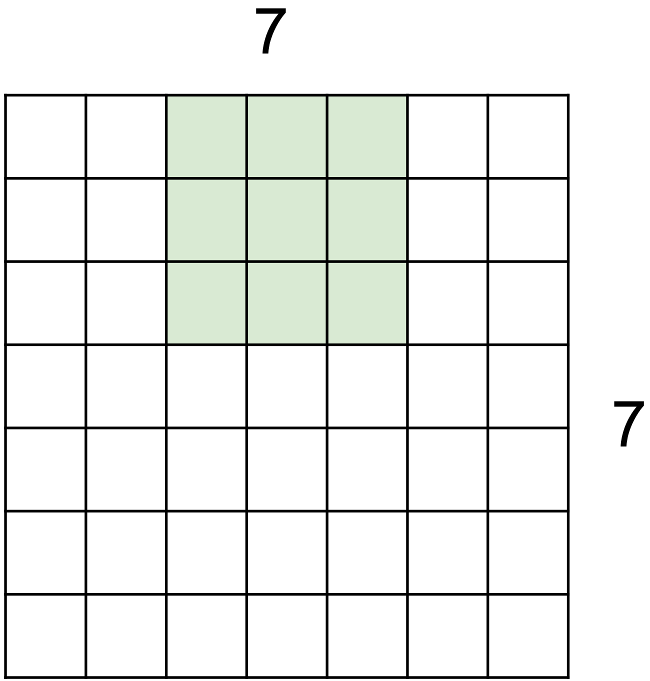
N = input dimension
P = padding size
F = filter size
Output size = (N – F + 2P) / stride + 1
= (7 – 3 + 2 * 1) / 1 + 1 = 7

A closer look at spatial dimensions:



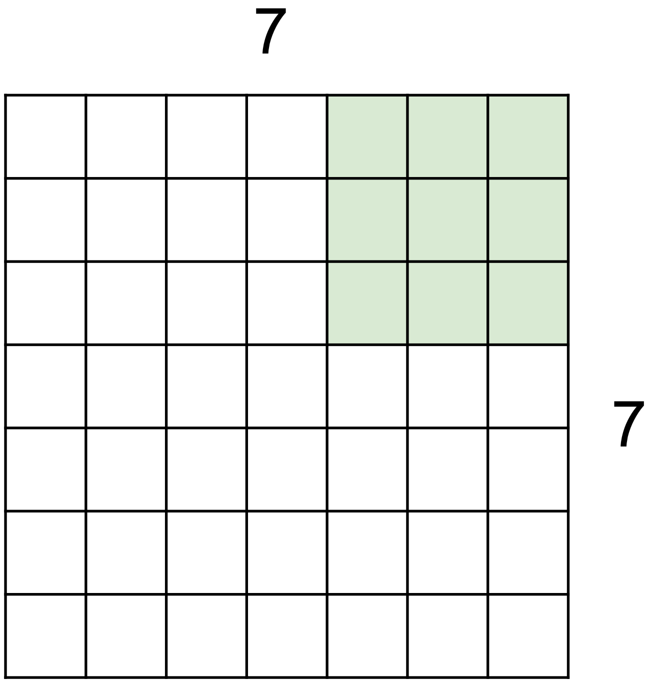
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



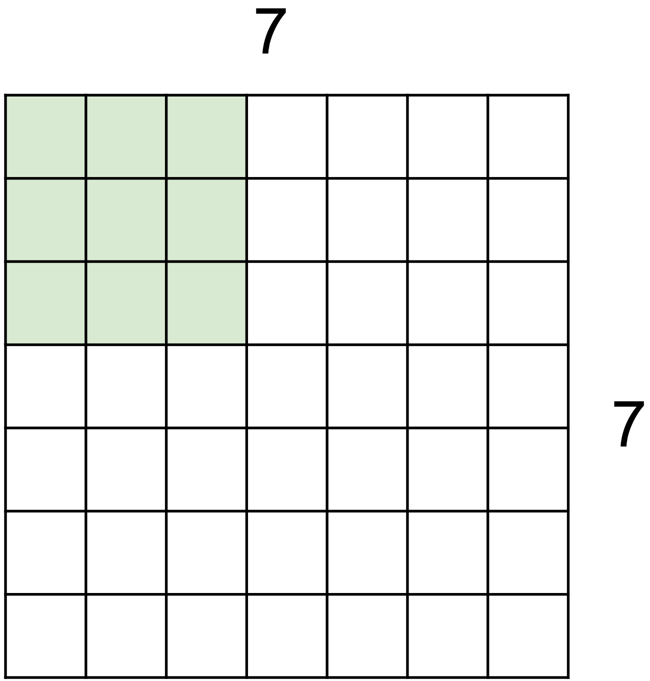
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**

A closer look at spatial dimensions:



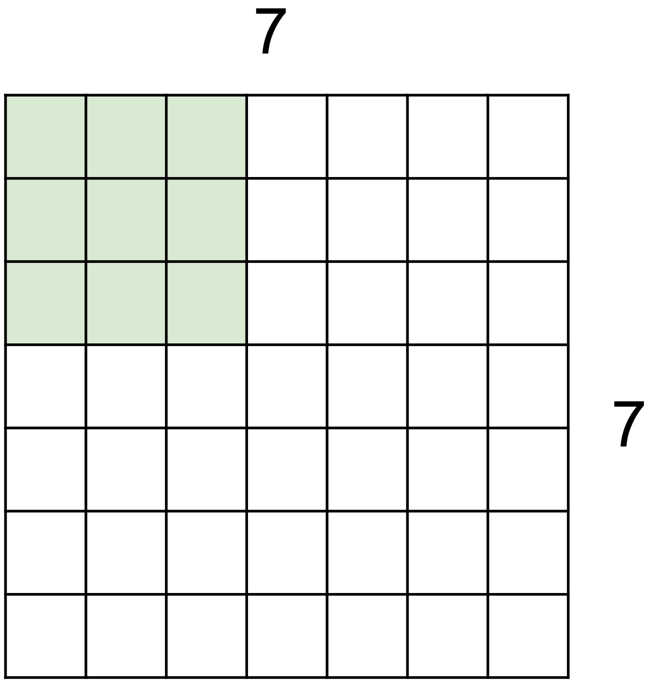
7x7 input (spatially)
assume 3x3 filter
applied **with stride 2**
=> 3x3 output!

A closer look at spatial dimensions:



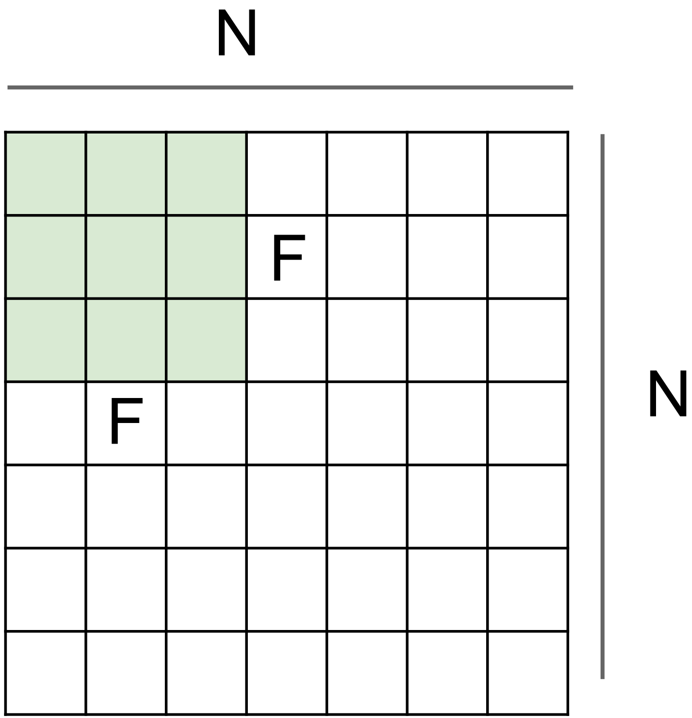
7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

A closer look at spatial dimensions:



7x7 input (spatially)
assume 3x3 filter
applied **with stride 3?**

doesn't fit!
cannot apply 3x3 filter on
7x7 input with stride 3.



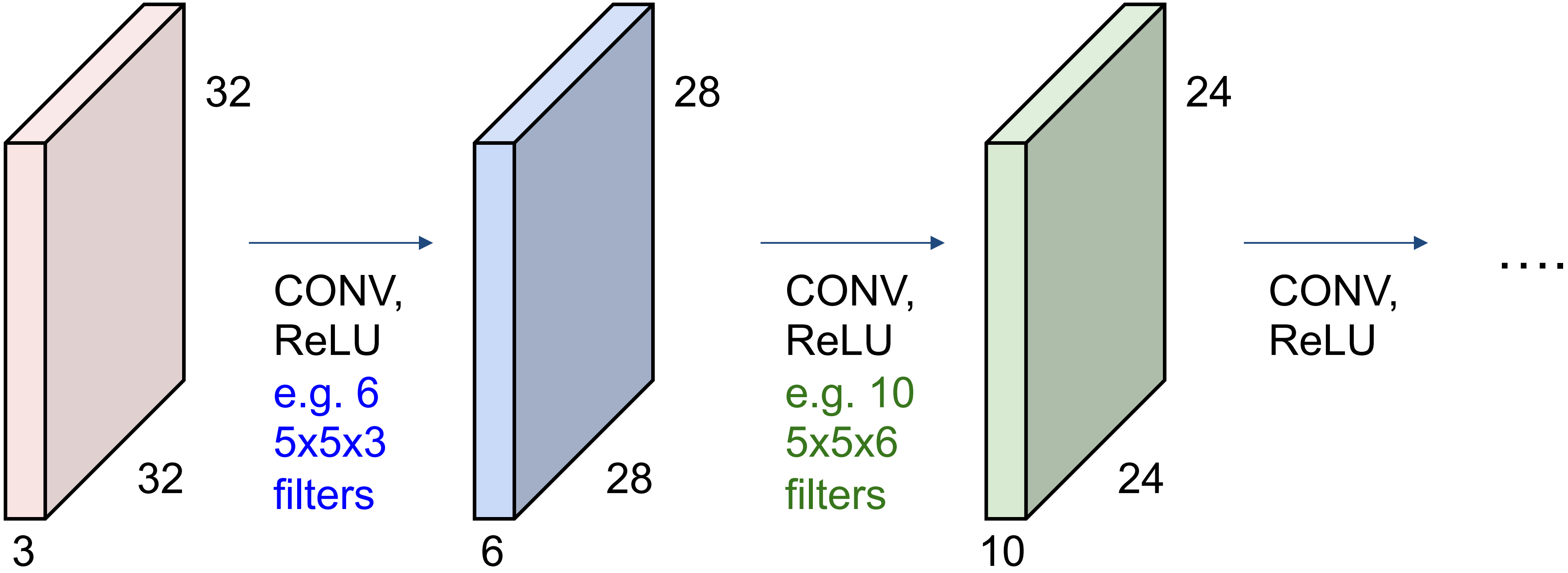
Output size:
(N - F) / stride + 1

e.g. N = 7, F = 3:
stride 1 => (7 - 3)/1 + 1 = 5
stride 2 => (7 - 3)/2 + 1 = 3
stride 3 => (7 - 3)/3 + 1 = 2.33 :\

With padding of 1 x 1:
stride 3 => (7 – 3 **+ 2**)/3 + 1 = 3

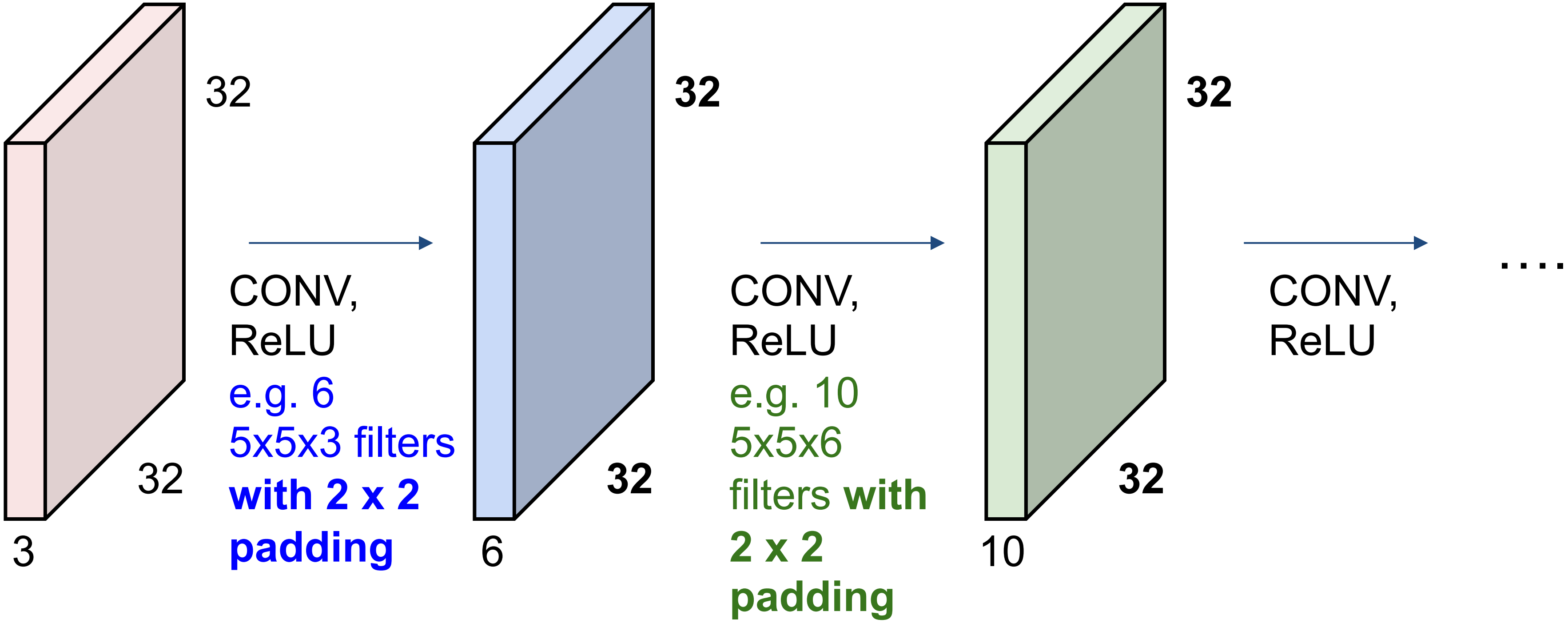
Remember back to…

E.g. 32x32 input convolved repeatedly with 5x5 filters shrinks volumes spatially! (32 -> 28 -> 24 ...). Shrinking too fast is not good, doesn't work well.

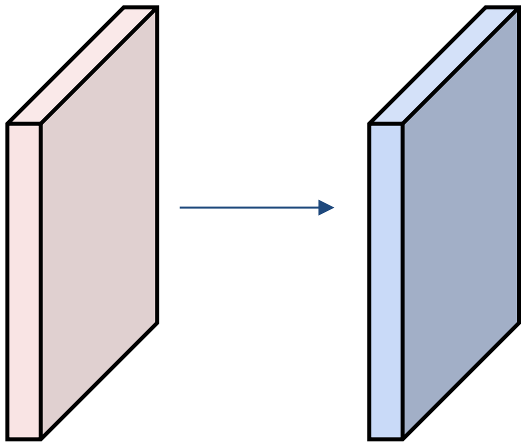


Remember back to…

With padding, we can keep the same spatial feature dimension throughout the convolution layers.



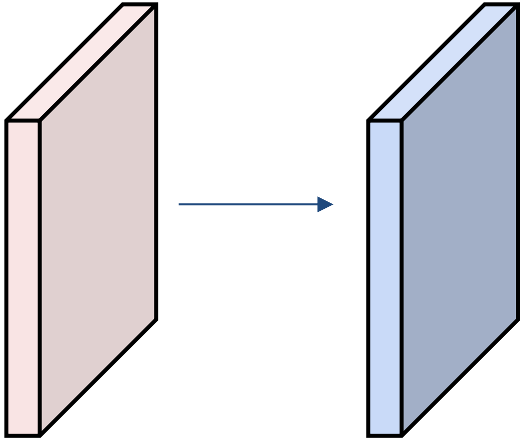
Examples time:



Input volume: **32x32x3**
Conv layer: 10 5x5 filters with stride 1, pad 2

Output volume size: ?

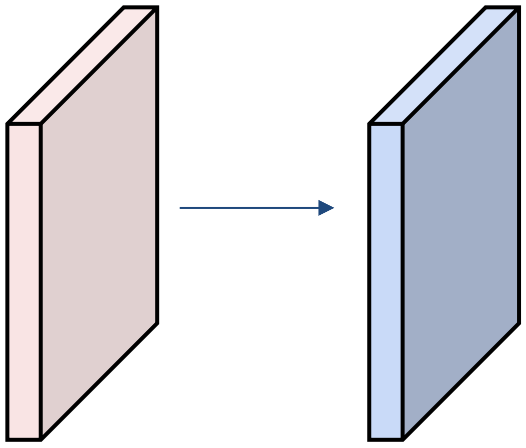
Examples time:



Input volume: **32x32x3**
Conv layer: 10 5x5 filters with stride 1, pad 2

Output volume size:
$(32+2*2-5)/1+1 = 32$ spatially, so
32x32x10

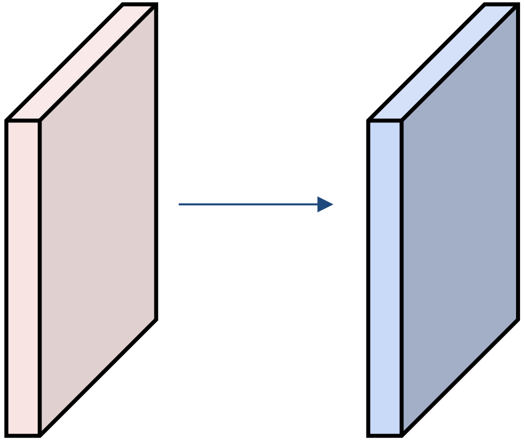
Examples time:



Input volume: **32x32x3**
Conv layer: 10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

Examples time:



Input volume: **32x32x3**

10 5x5 filters with stride 1, pad 2

Number of parameters in this layer?

each filter has 5*5*3 + 1 = 76 params (+1 for bias)

=> 76*10 = **760**

Convolution layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of W_2 x H_2 x K
where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

Number of parameters: F^2CK and K biases

Convolution layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

This will produce an output of W_2 x H_2 x K
where:

- $W_2 = (W_1 - F + 2P)/S + 1$
- $H_2 = (H_1 - F + 2P)/S + 1$

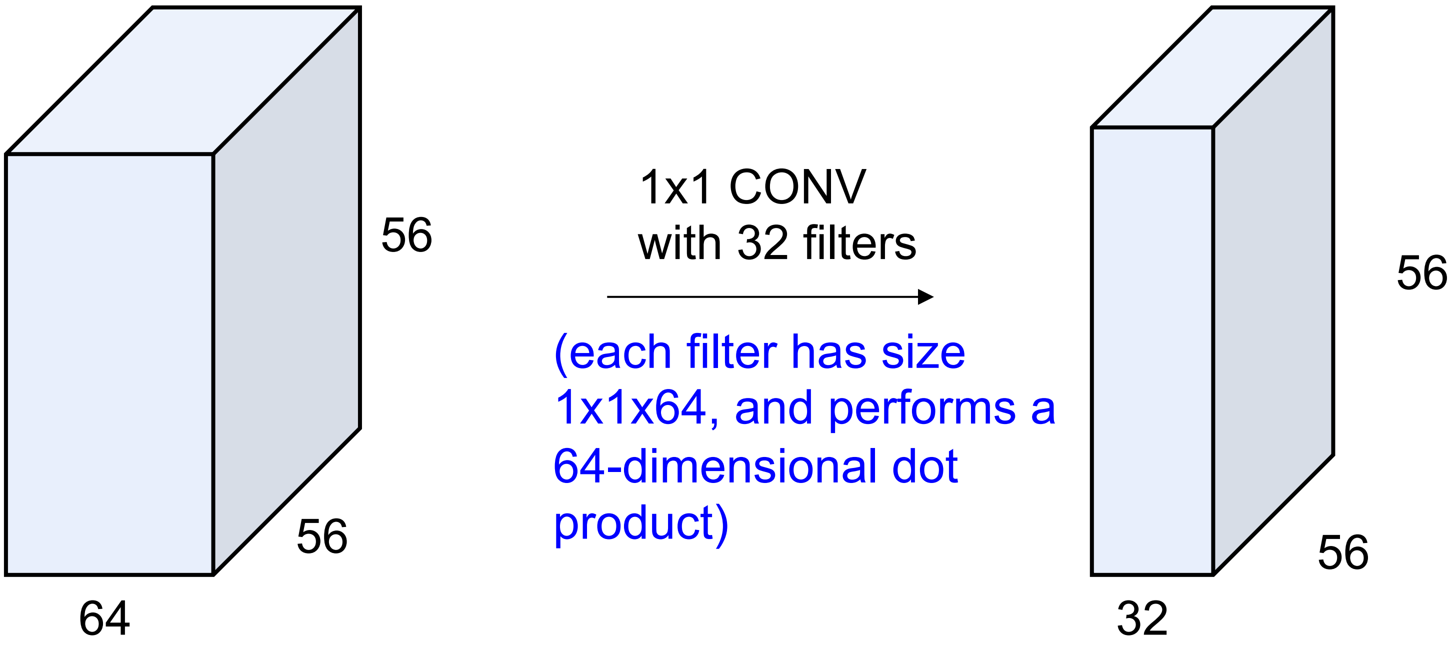
Number of parameters: F^2CK and K biases

Common settings:

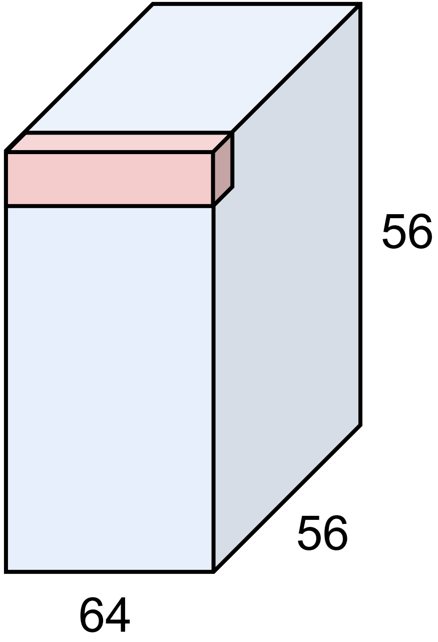
K = (powers of 2, e.g. 32, 64, 128, 512

- F = 3, S = 1, P = 1
- F = 5, S = 1, P = 2
- F = 5, S = 2, P = ? (whatever fits)
- F = 1, S = 1, P = 0

(btw, 1x1 convolution layers make perfect sense)



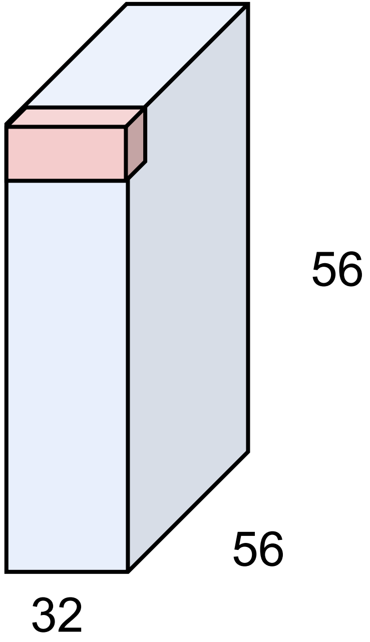
(btw, 1x1 convolution layers make perfect sense)



1x1 CONV
with 32 filters

(each filter has size 1x1x64, and performs a 64-dimensional dot product)

Grows or shrinks feature channel dimension



Example: CONV layer in PyTorch

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True)` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

where $\star$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

- `stride` controls the stride for the cross-correlation, a single number or a tuple.
- `padding` controls the amount of implicit zero-paddings on both sides for `padding` number of points for each dimension.
- `dilation` controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this link has a nice visualization of what `dilation` does.
- `groups` controls the connections between inputs and outputs. `in_channels` and `out_channels` must both be divisible by `groups`. For example,
  - At groups=1, all inputs are convolved to all outputs.
  - At groups=2, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
  - At groups= `in_channels`, each input channel is convolved with its own set of filters, of size: $\left\lfloor \frac{C_{\text{out}}}{C_{\text{in}}} \right\rfloor$.

The parameters `kernel_size`, `stride`, `padding`, `dilation` can either be:

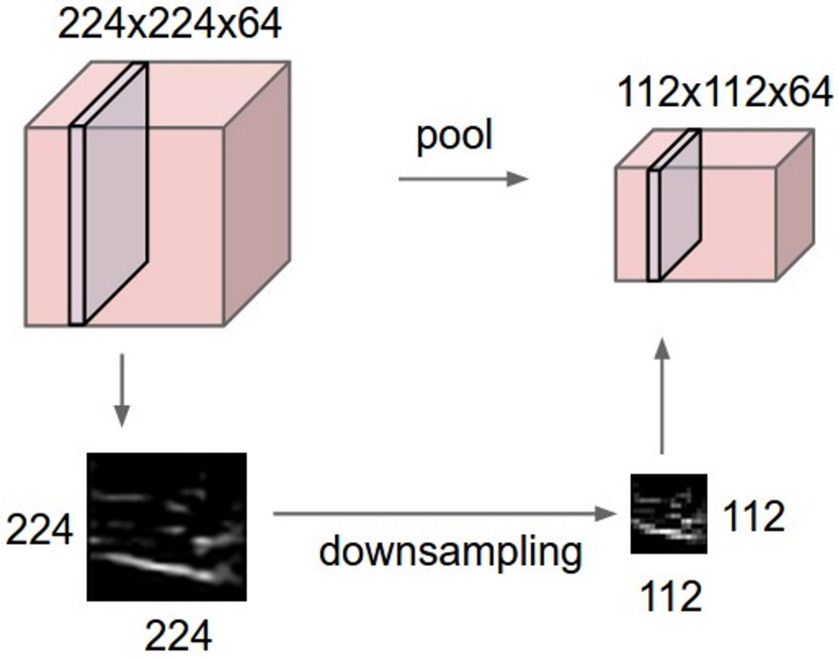
- a single `int` – in which case the same value is used for the height and width dimension
- a `tuple` of two ints – in which case, the first *int* is used for the height dimension, and the second *int* for the width dimension

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

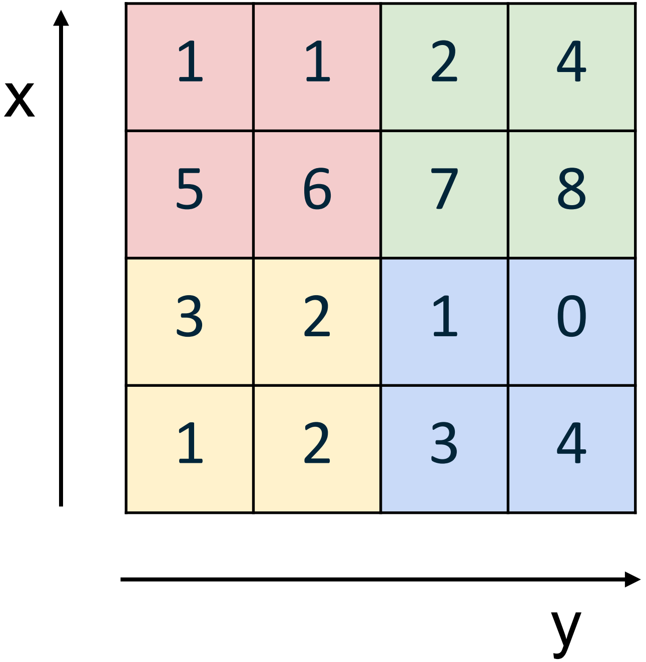
Pooling layer (down-sampling)

- makes the representations spatially smaller
- saves computation (GPU mem & speed), allows go deeper
- operates over each activation map independently:

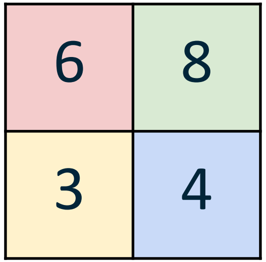


MAX POOLING

Single depth slice



max pool with 2x2 filters
and stride 2



- Intuitively, only forward the most important features in the region.
- Also improve spatial invariance (output is agnostic to where the max value comes from)

Pooling layer: summary

Let's assume input is W_1 x H_1 x C
Conv layer needs 2 hyperparameters:

- The spatial extent **F**
- The stride **S**

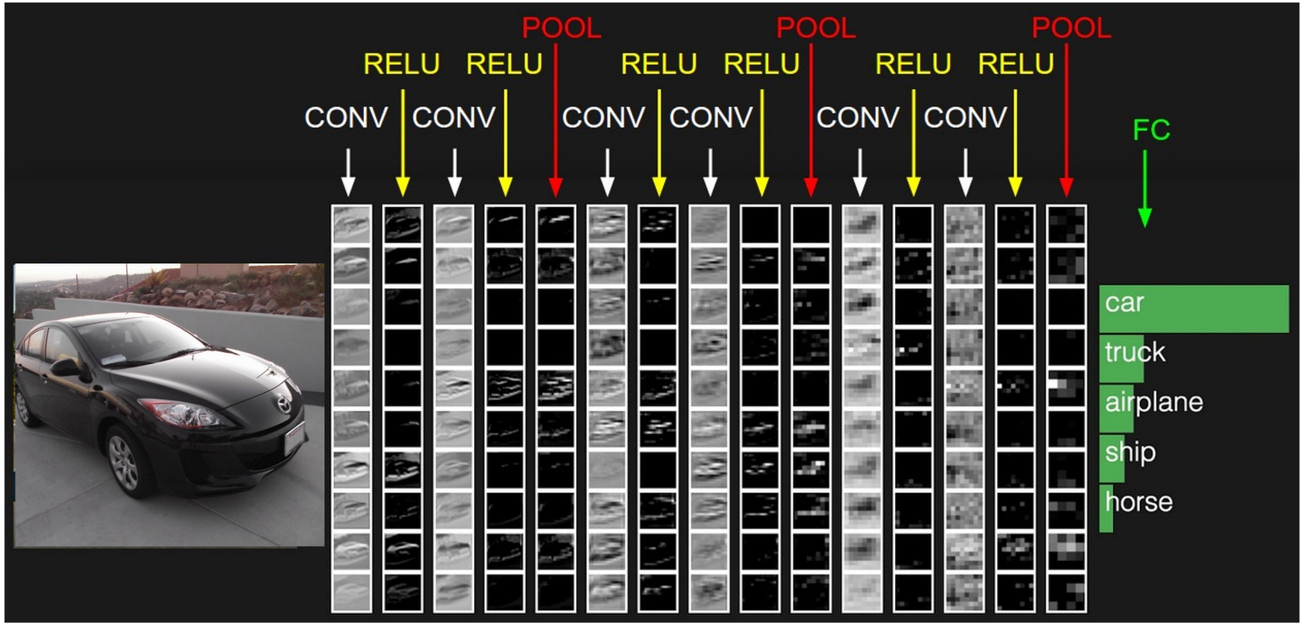
This will produce an output of W_2 x H_2 x C where:

- $W_2 = (W_1 - F)/S + 1$
- $H_2 = (H_1 - F)/S + 1$

Number of parameters: 0

Fully Connected Layer (FC layer)

- After flattening convolution feature maps to 1-D vector



Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Historically architectures looked like

  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX

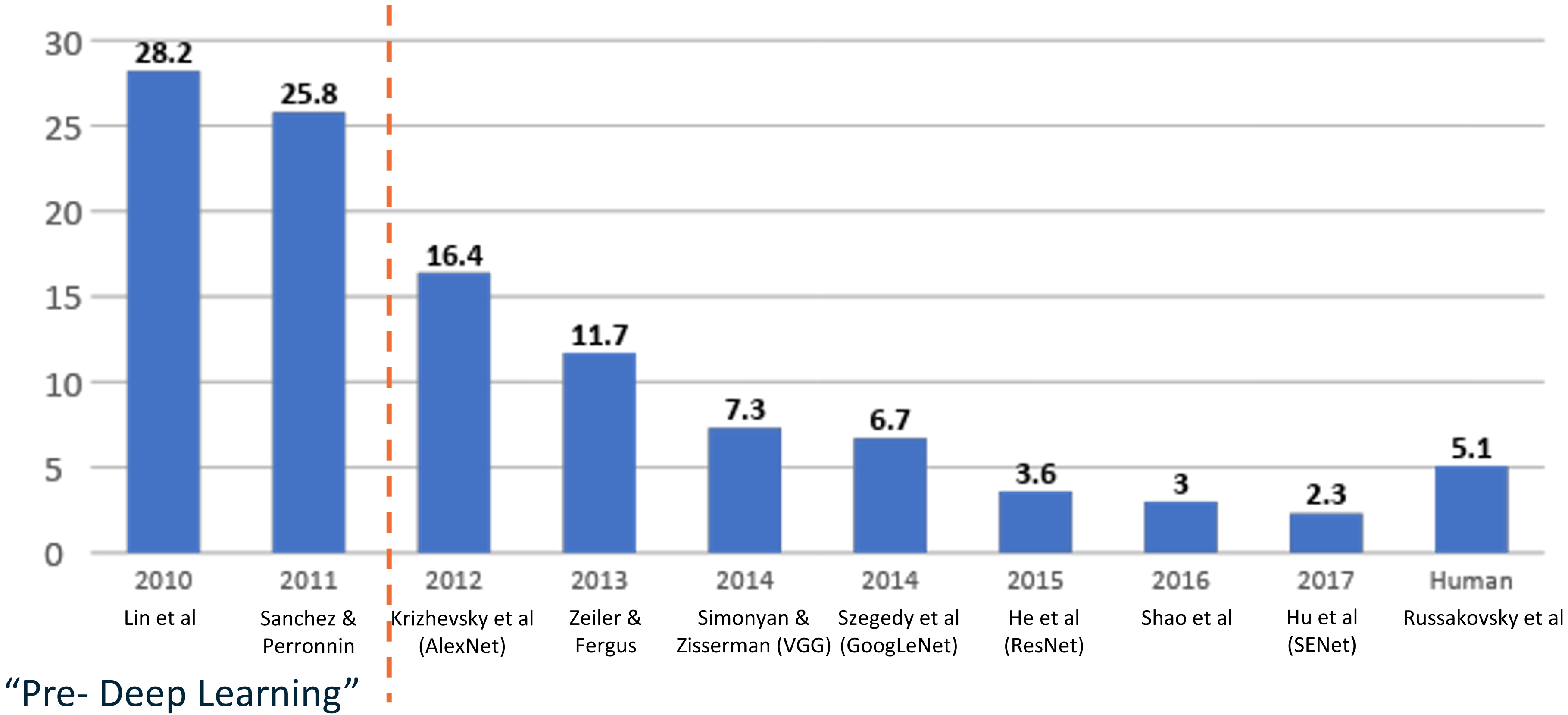
  where N is usually up to ~5, M is large, 0 <= K <= 2.

  - but recent advances such as ResNet/ViT have changed this paradigm

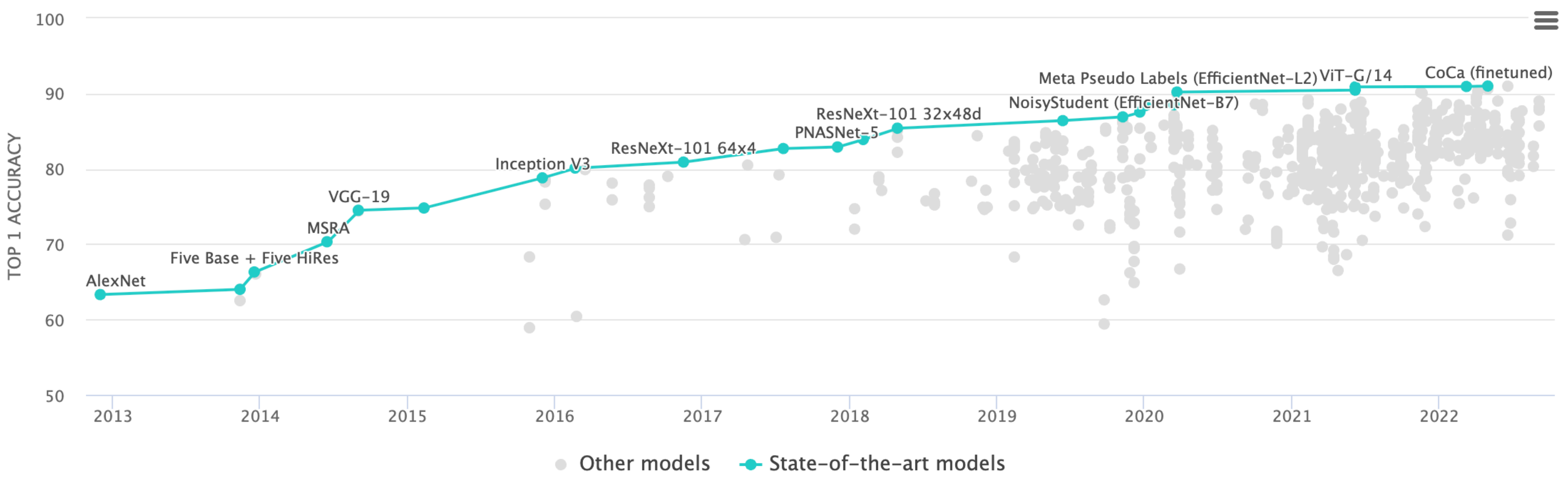
ConvNets: Where are we today?

The **ImageNet** dataset contains 14,197,122 annotated images according to the WordNet hierarchy. ImageNet Large Scale Visual Recognition Challenge (ILSVRC) is a benchmark for image classification and object detection based on the dataset.

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



ConvNets: Where are we today?



CNN Architectures

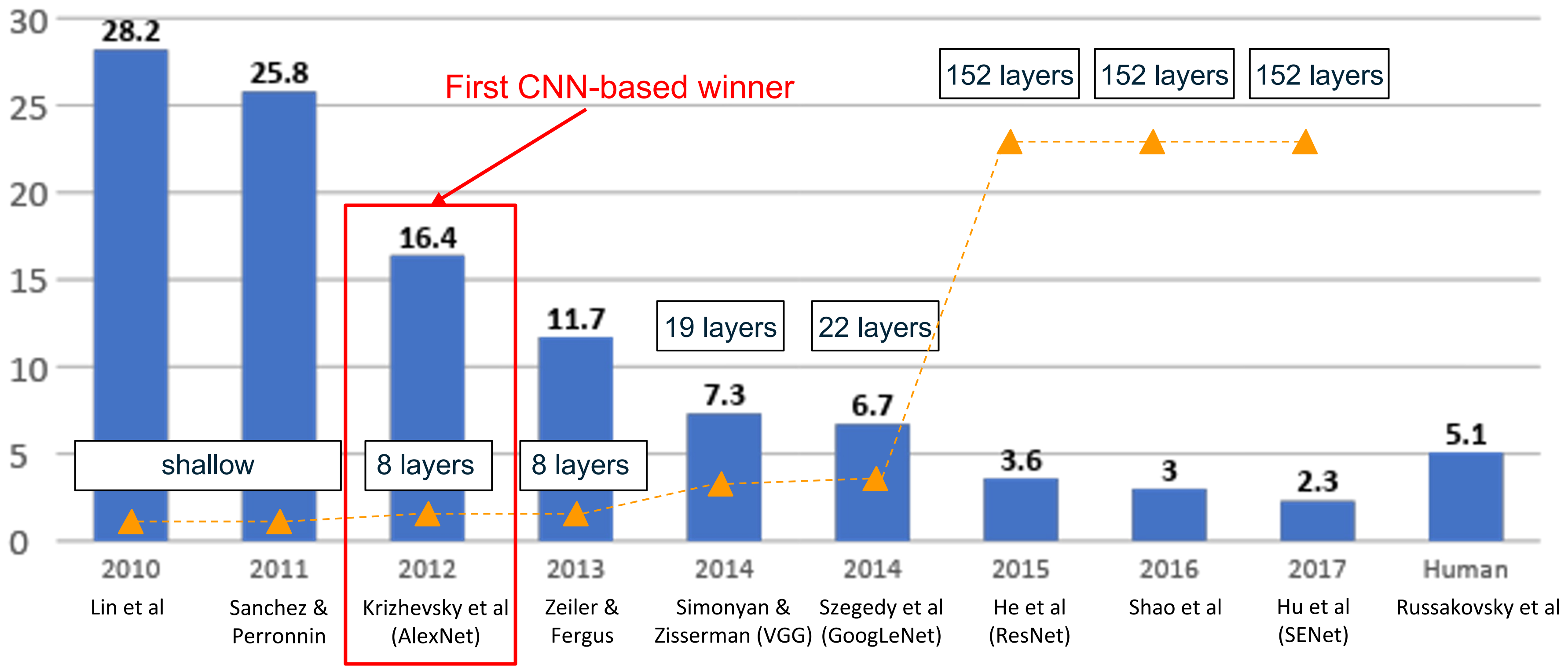
Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet

Also....

- SENet
- Wide ResNet
- ResNeXT
- DenseNet
- MobileNets
- NASNet
- EfficientNet

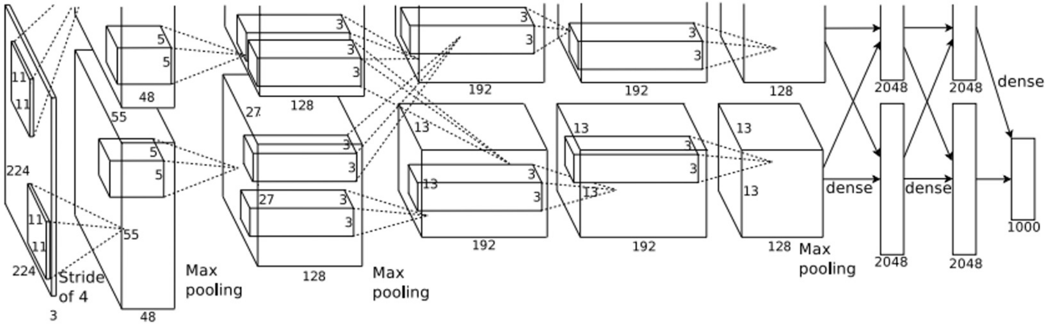
ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



Case Study: AlexNet

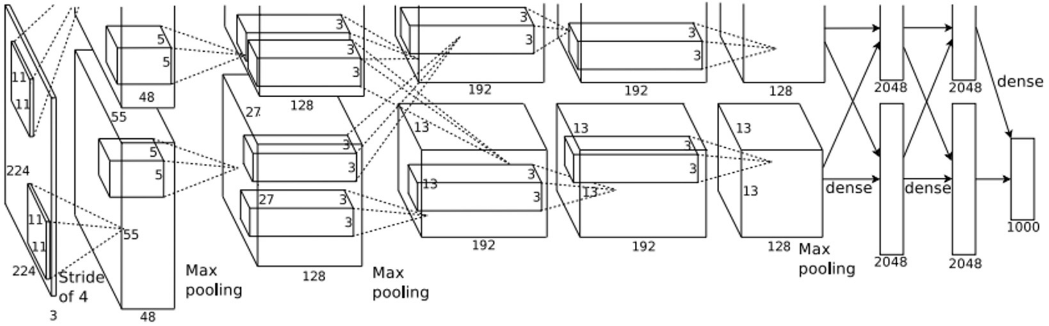
[Krizhevsky et al. 2012]

Architecture:
CONV1
MAX POOL1
NORM1
CONV2
MAX POOL2
NORM2
CONV3
CONV4
CONV5
Max POOL3
FC6
FC7
FC8



Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

W’ = (W - F + 2P) / S + 1

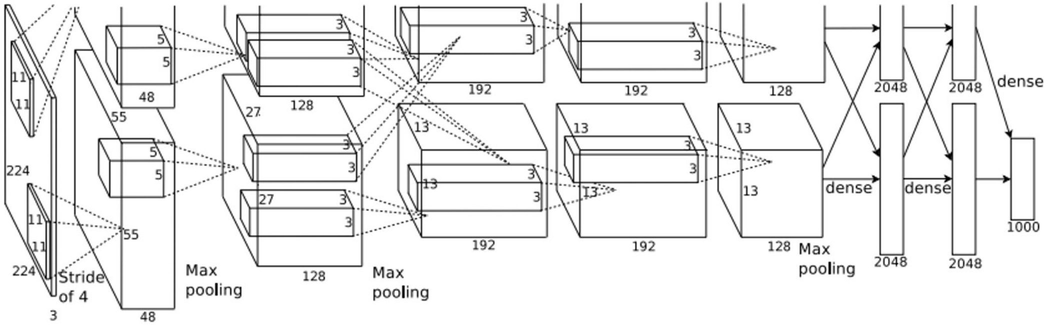
First layer (CONV1): 96 11x11 filters applied at stride 4

=>

Q: what is the output volume size? Hint: (227-11)/4+1 = 55

Case Study: AlexNet

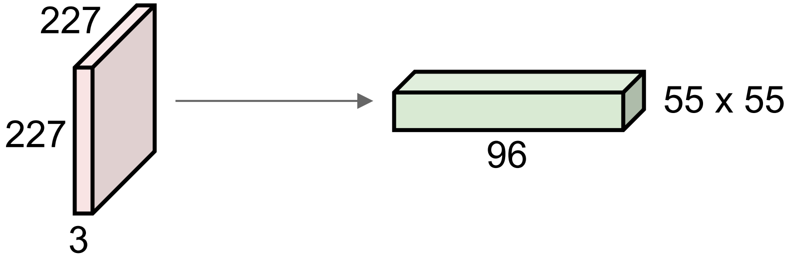
[Krizhevsky et al. 2012]



Input: 227x227x3 images

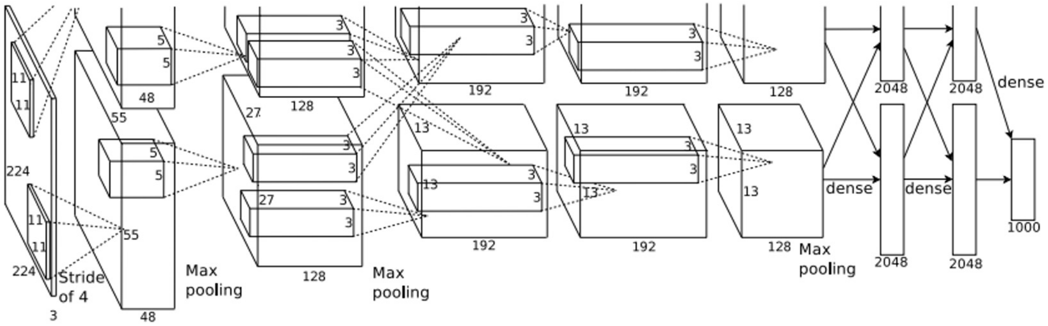
First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**

W' = (W - F + 2P) / S + 1



Case Study: AlexNet

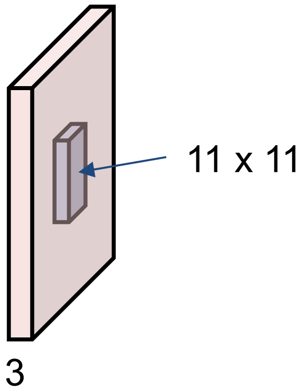
[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?



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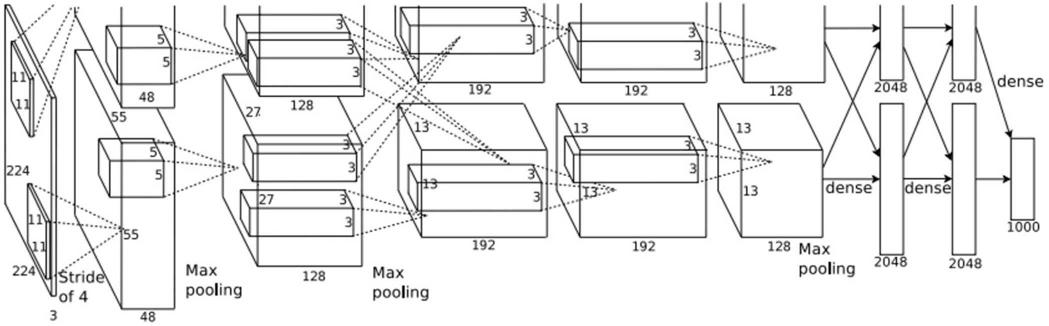
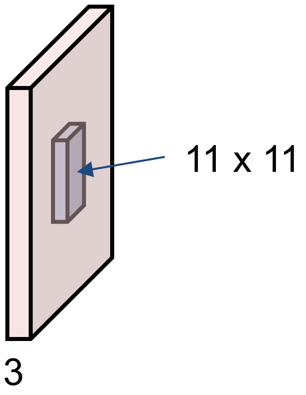
Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**
Parameters: (11*11*3 + 1)*96 = **35K**



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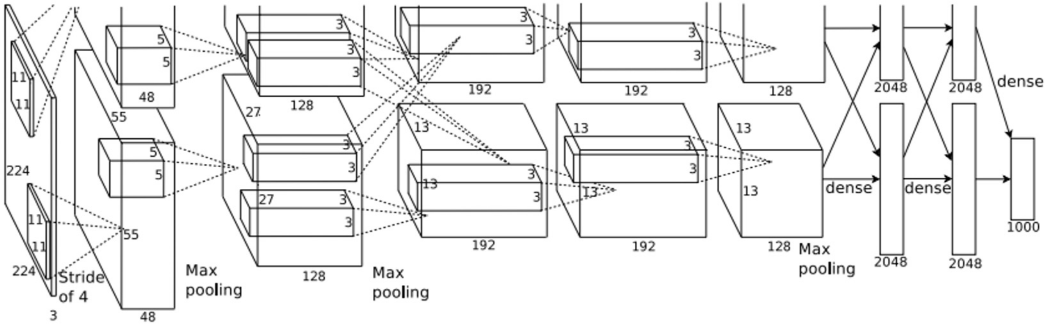
Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images
After CONV1: 55x55x96

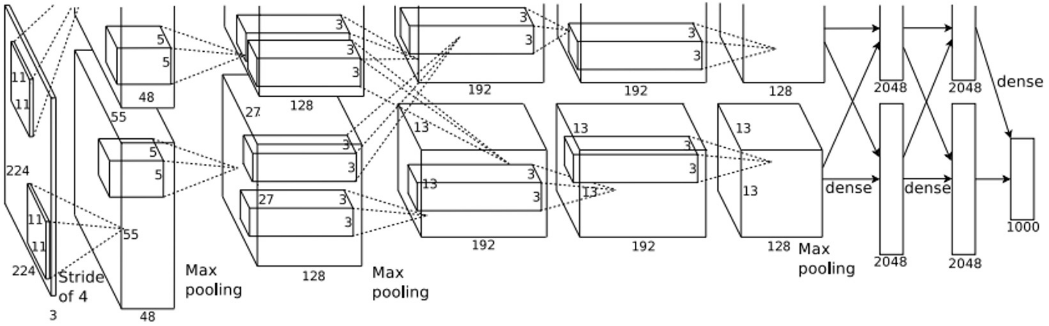
W’ = (W - F + 2P) / S + 1

Second layer (POOL1): 3x3 filters applied at stride 2

Q: what is the output volume size? Hint: (55-3)/2+1 = 27

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images
After CONV1: 55x55x96

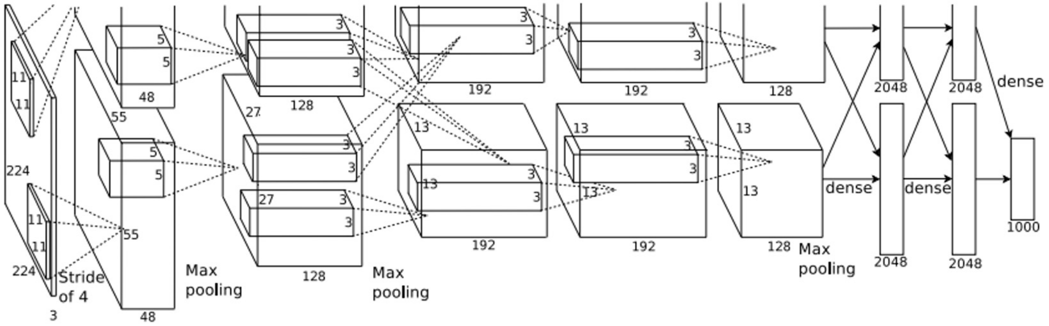
$$W' = (W - F + 2P) / S + 1$$

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]

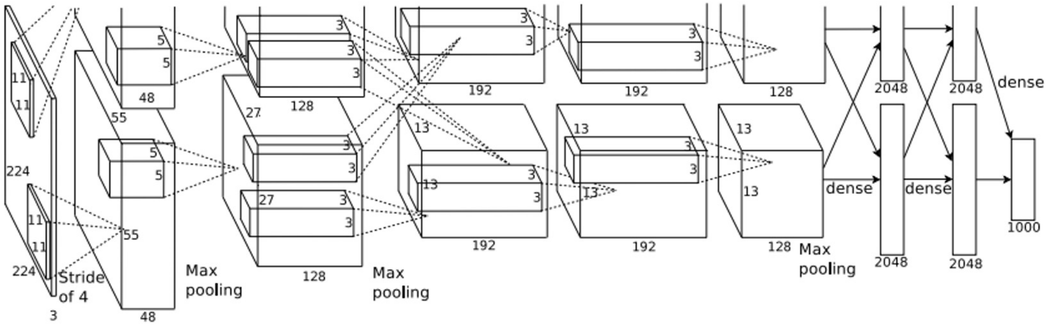


Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96
Parameters: 0!

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images
After CONV1: 55x55x96
After POOL1: 27x27x96
...

Case Study: AlexNet

[Krizhevsky et al. 2012]

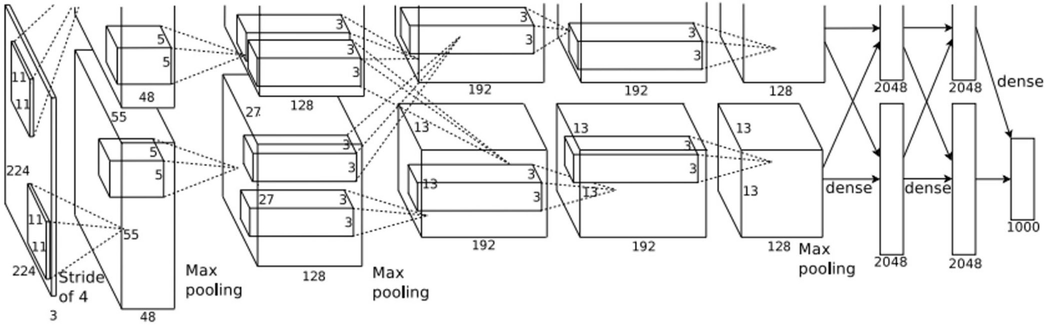
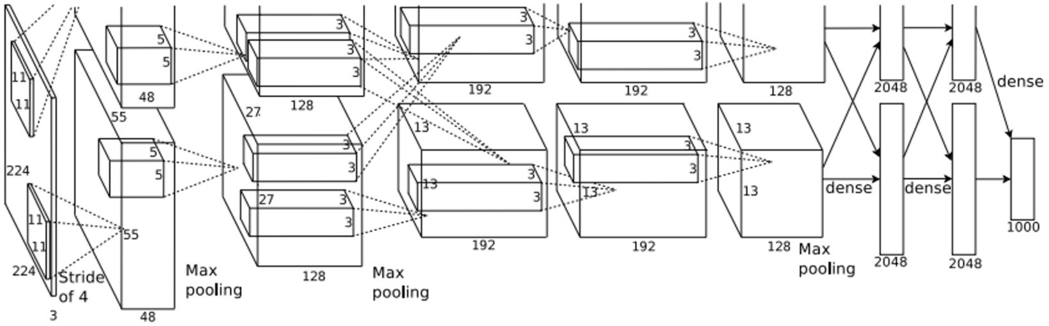


Figure copyright Alex Krizhevsky, Ilya Sutskever, and Geoffrey Hinton, 2012. Reproduced with permission.

Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Case Study: AlexNet

[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
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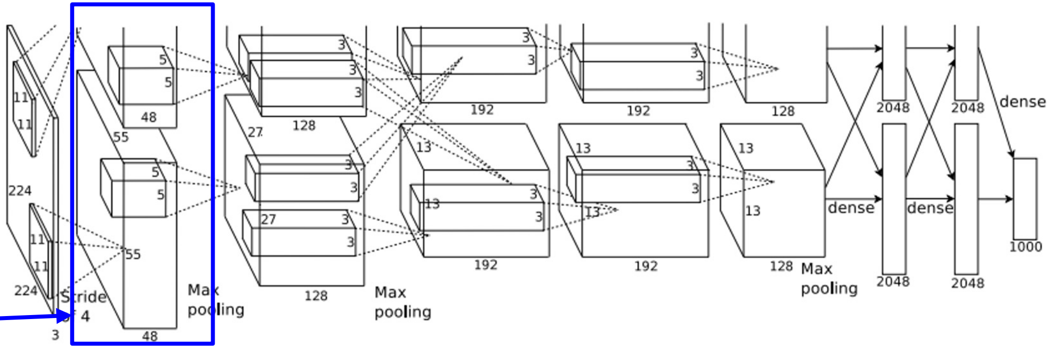
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:
- first use of ReLU
- used Norm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

Case Study: AlexNet

[Krizhevsky et al. 2012]



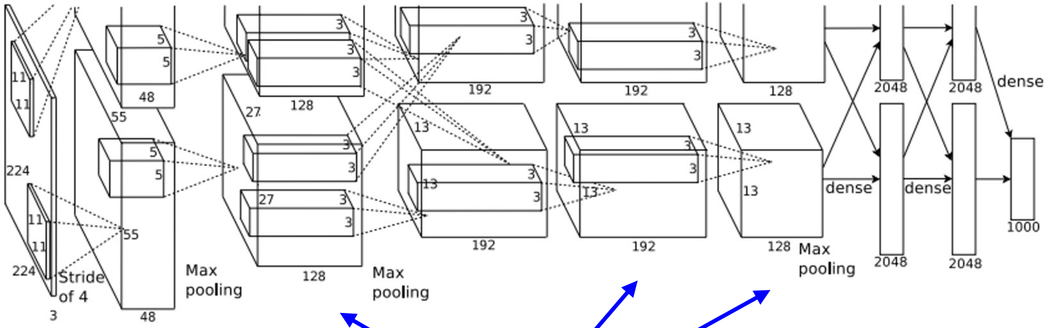
[55x55x48] x 2

Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Historical note: Trained on GTX 580 GPU with only 3 GB of memory. Network spread across 2 GPUs, half the neurons (feature maps) on each GPU.

Case Study: AlexNet

[Krizhevsky et al. 2012]



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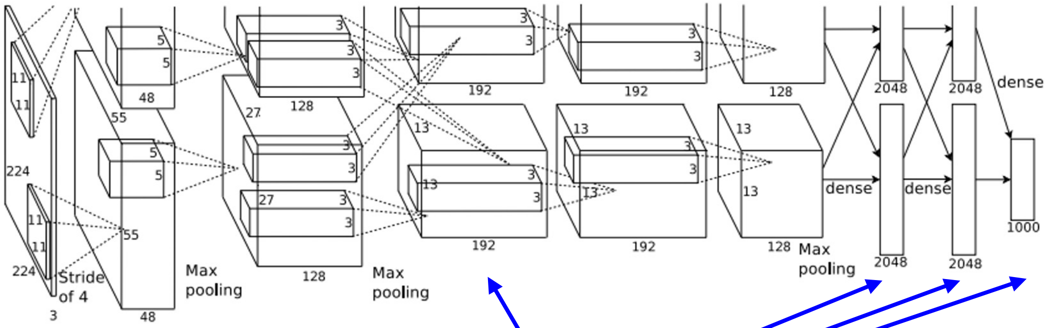
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

CONV1, CONV2, CONV4, CONV5:
Connections only with feature maps on same GPU

Case Study: AlexNet

[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
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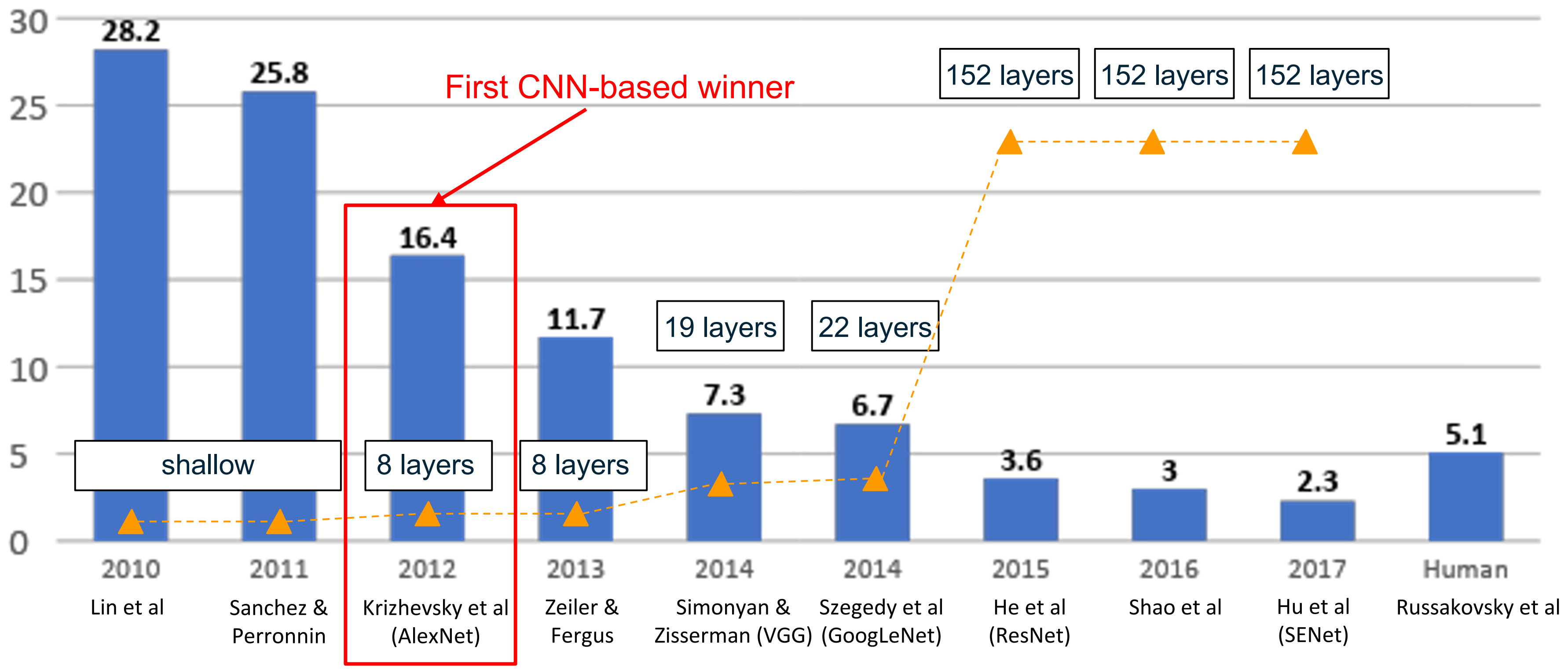
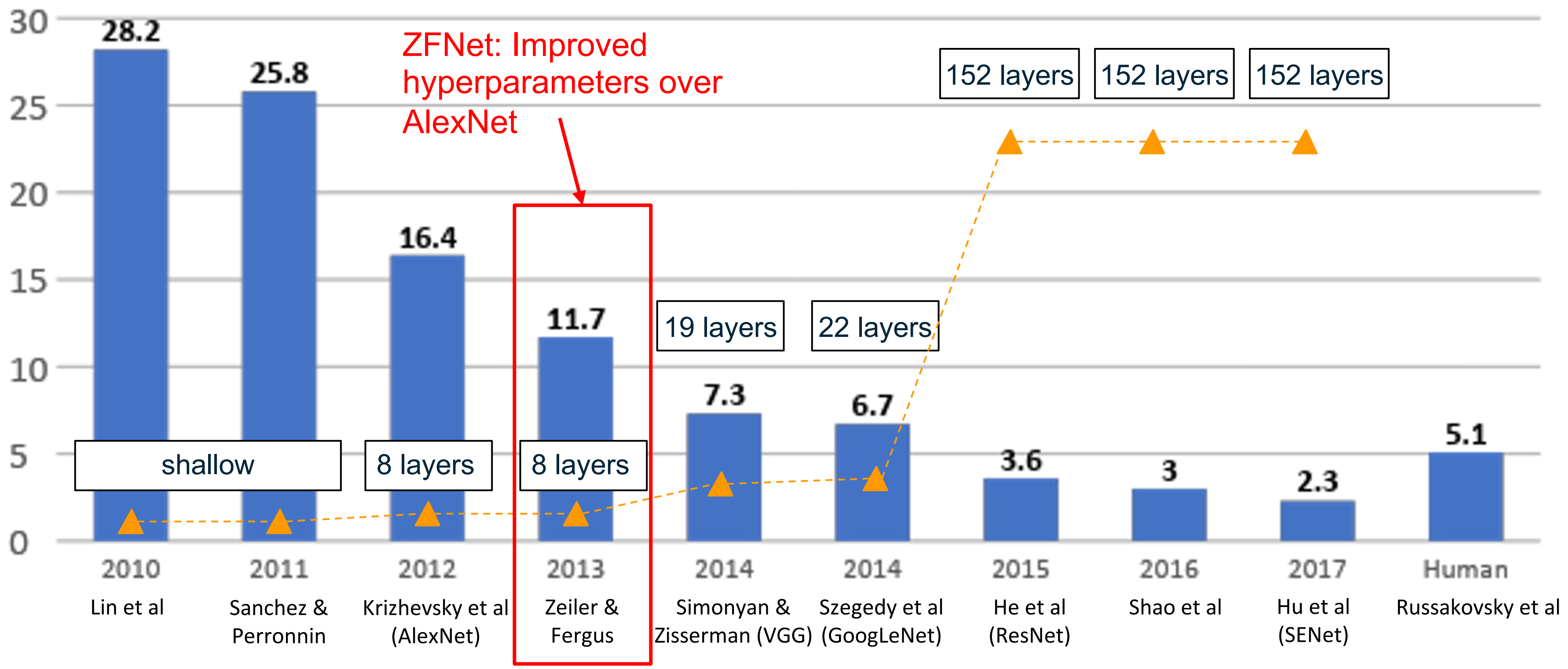
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

CONV3, FC6, FC7, FC8:
Connections with all feature maps in preceding layer, communication across GPUs

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners

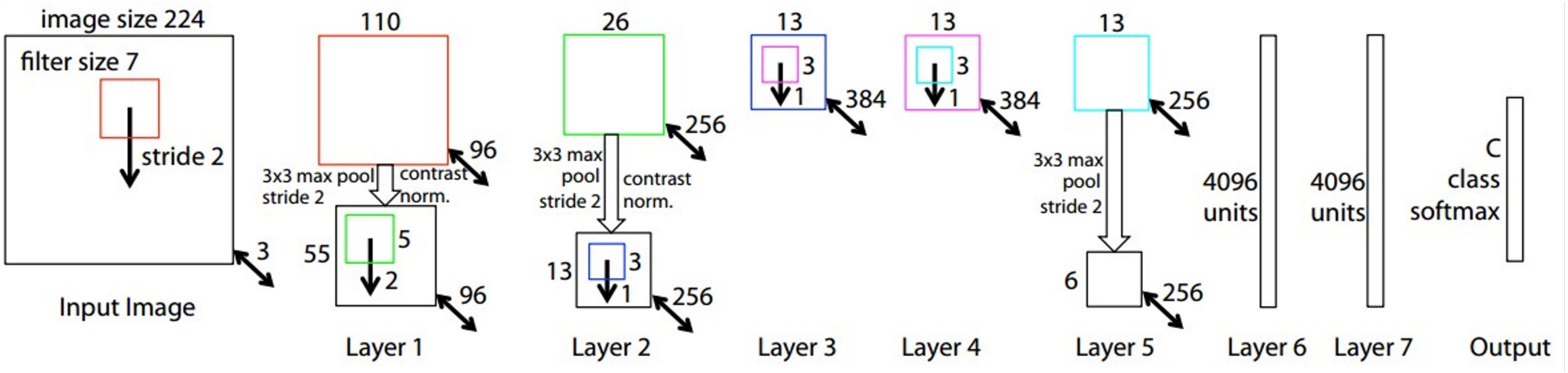


ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



ZFNet

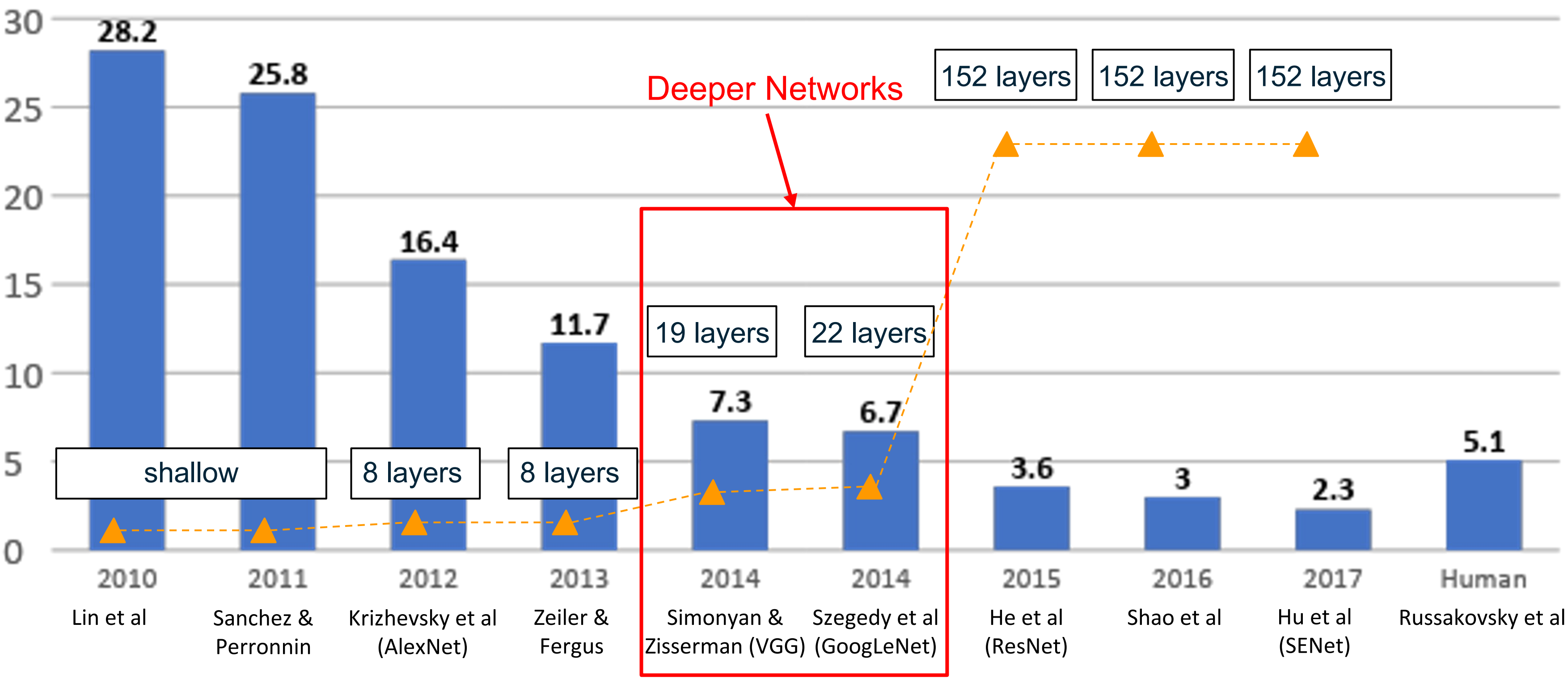
[Zeiler and Fergus, 2013]



AlexNet but:
CONV1: change from (11x11 stride 4) to (7x7 stride 2)
CONV3,4,5: instead of 384, 384, 256 filters use 512, 1024, 512

ImageNet top 5 error: 16.4% -> 11.7%

ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



Next time: More CNN Architectures!

Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet

Also....

- SENet
- Wide ResNet
- ResNeXT
- DenseNet
- MobileNets
- NASNet
- EfficientNet