

# **CS 4644-DL / 7643-A: LECTURE 9**

## **DANFEI XU**

Topics:

- Convolutional Neural Networks Architectures (cont.)
- Training Neural Networks (Part 1)

# Administrative

- PS1/HW1 due **today** (grace period till Sep 21<sup>st</sup>)
- PS2/HW2 out: **Difficult assignment. Start early!**
- Project proposal due **Sep 26<sup>th</sup>. No extension!**

# CNN Architectures

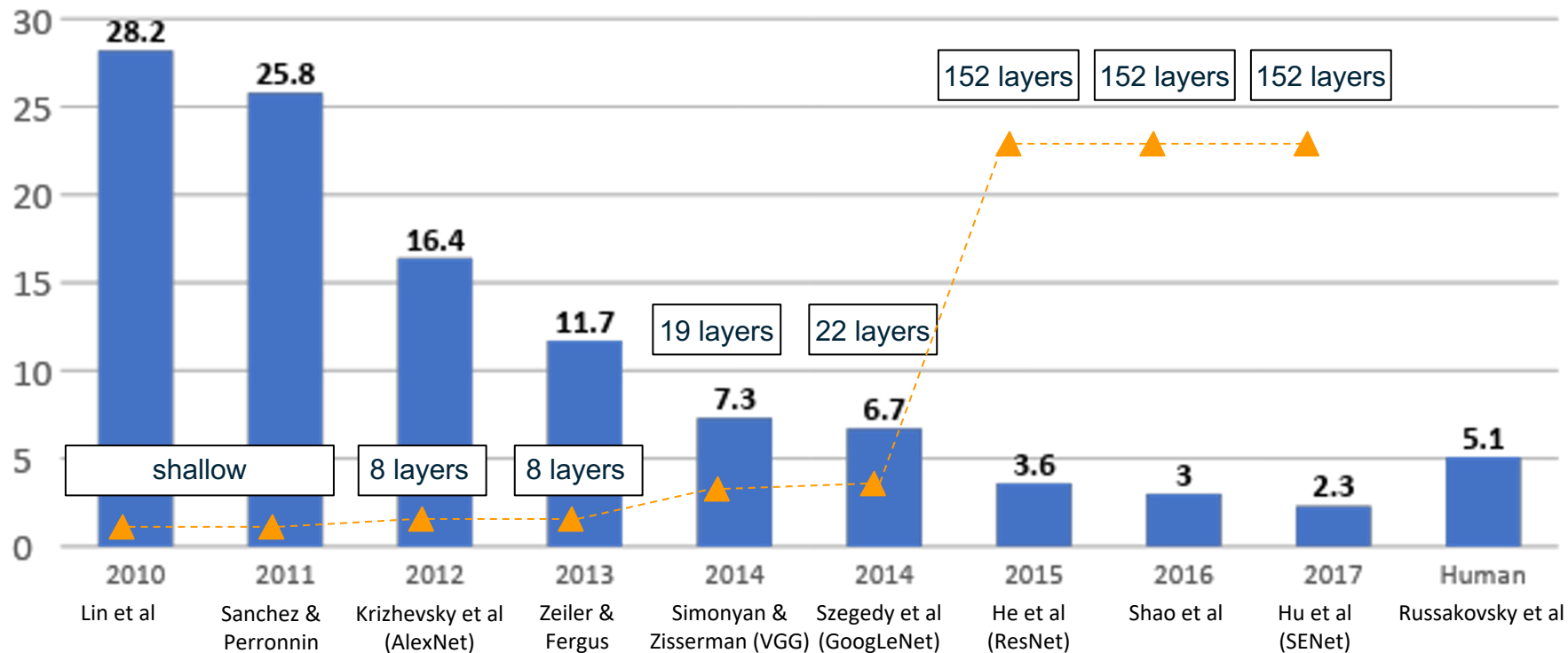
## Case Studies

- AlexNet
- VGG
- GoogLeNet
- ResNet

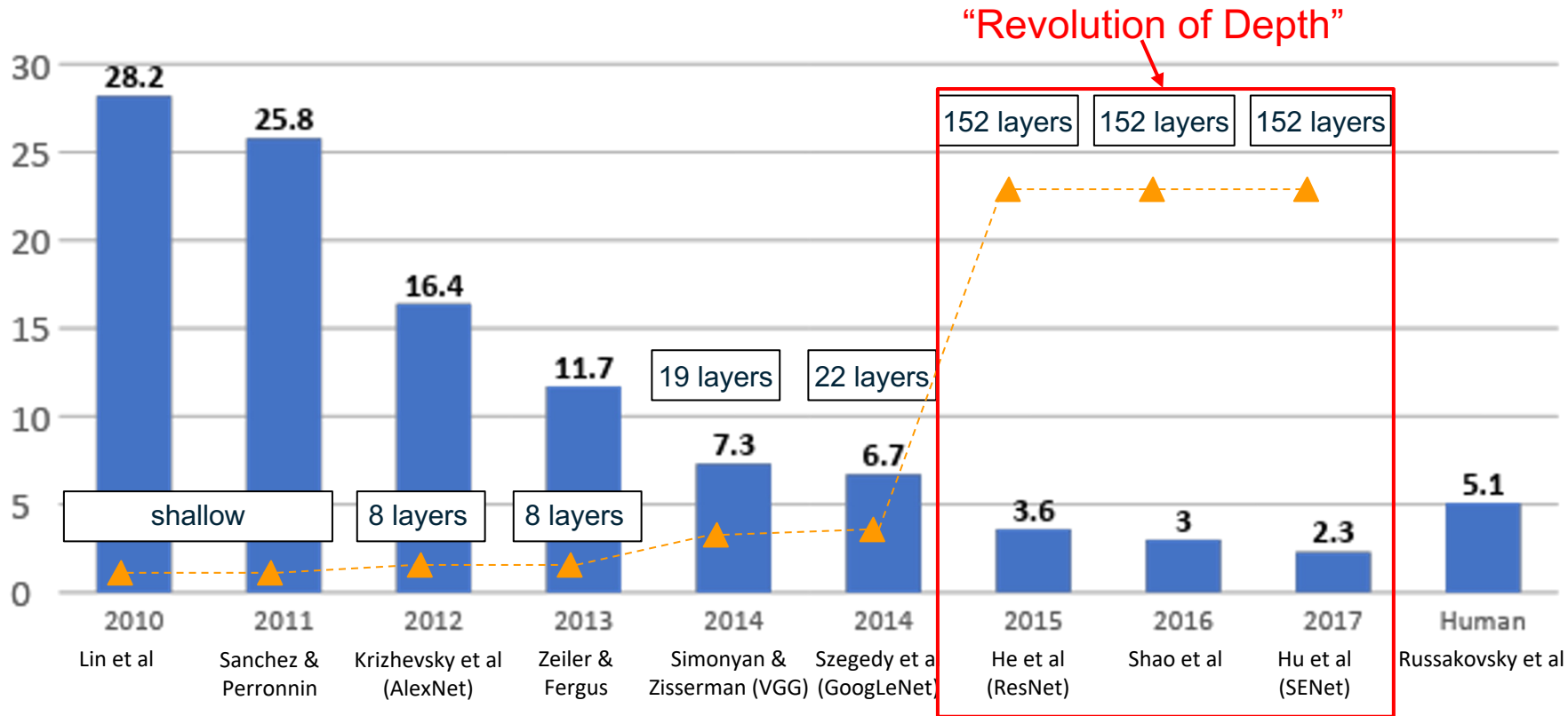
## Also.....

- SENet
- Wide ResNet
- ResNeXT
- DenseNet
- MobileNets
- NASNet
- EfficientNet

# ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



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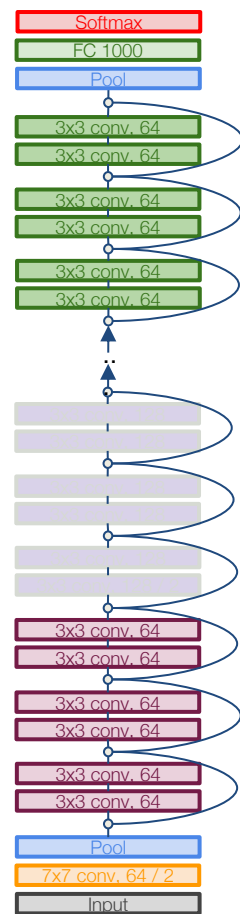
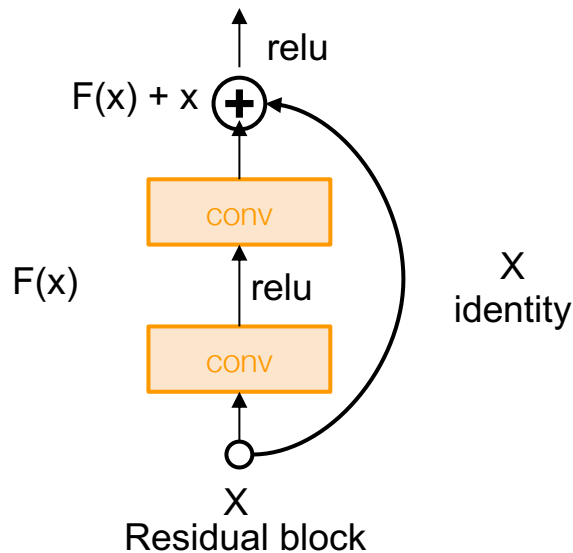


# Case Study: ResNet

[He et al., 2015]

Very deep networks using residual connections

- 152-layer model for ImageNet
- ILSVRC'15 classification winner (3.57% top 5 error)
- Swept all classification and detection competitions in ILSVRC'15 and COCO'15!



# Case Study: ResNet

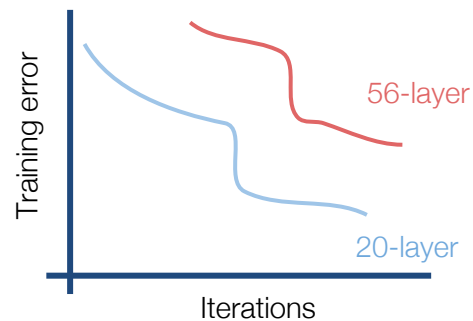
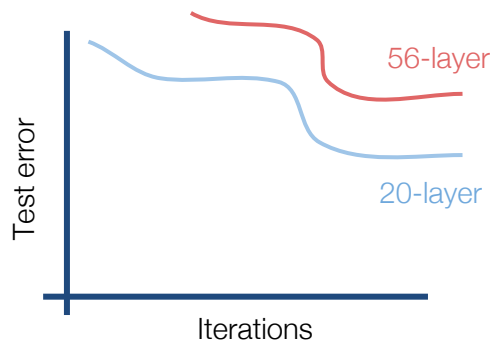
*[He et al., 2015]*

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?

# Case Study: ResNet

[He et al., 2015]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?

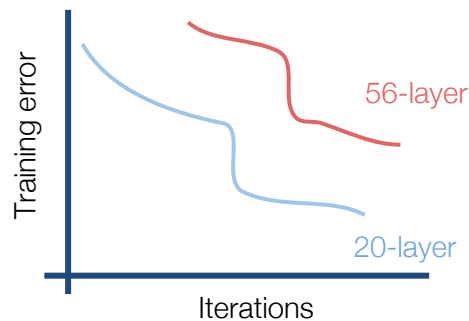
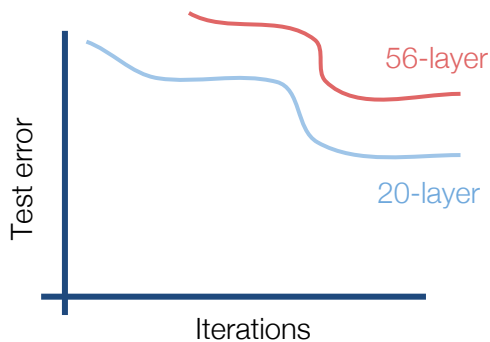




# Case Study: ResNet

[He et al., 2015]

What happens when we continue stacking deeper layers on a “plain” convolutional neural network?



56-layer model performs worse on both test and training error

-> The deeper model performs worse, but it's **not caused by overfitting!**

# Case Study: ResNet

[He et al., 2015]

Fact: Deep models have more representation power (more parameters) than shallower models.

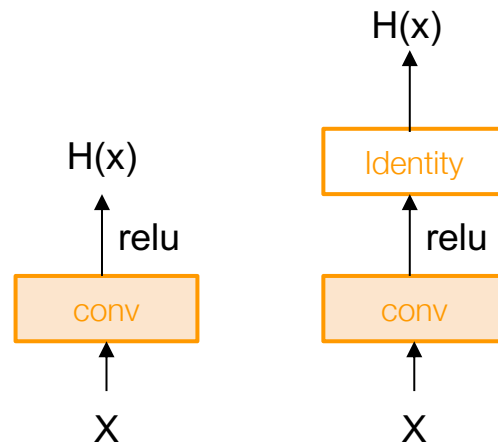
Hypothesis: the problem is an *optimization* problem,  
**deeper models are harder to optimize**

# Case Study: ResNet

[He et al., 2015]

A deeper model can **emulate** a shallower model: copy layers from shallower model, set extra layers to identity

Thus deeper models should do at least as good as shallow models



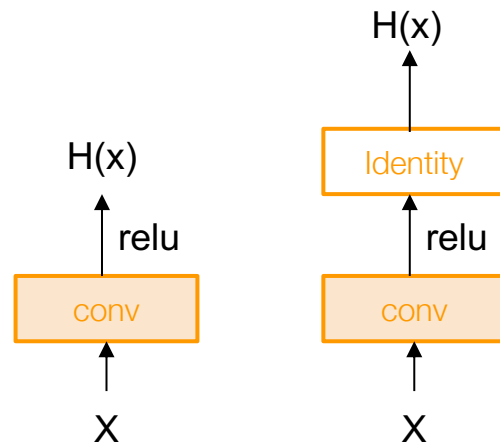
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Deeper models are harder to optimize. They don't learn identity functions (no-op) to emulate shallow models



# Case Study: ResNet

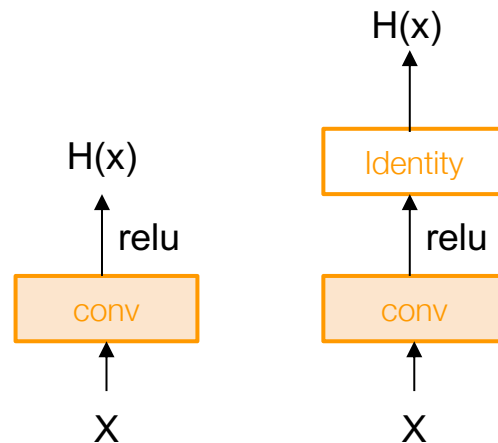
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A deeper model can **emulate** a shallower model: copy layers from shallower model, set extra layers to identity

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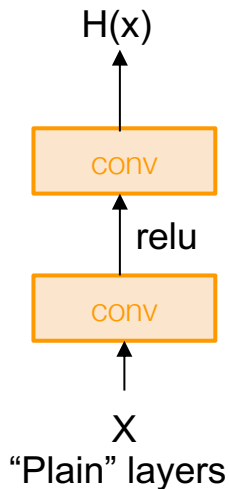
**Solution:** Change the network so learning identity functions (no-op) as extra layers is easy



# Case Study: ResNet

[He et al., 2015]

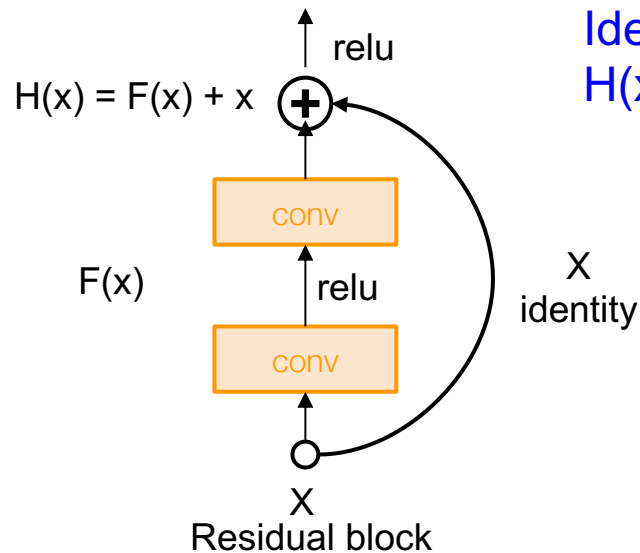
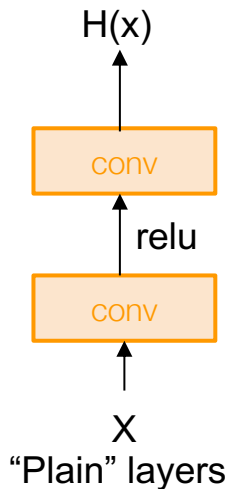
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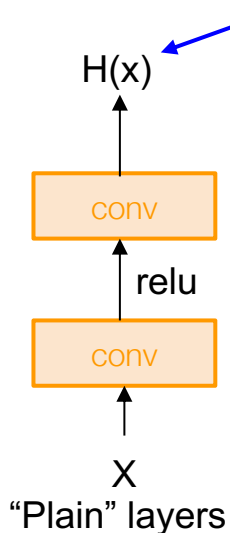


Identity mapping:  
 $H(x) = x$  if  $F(x) = 0$

# Case Study: ResNet

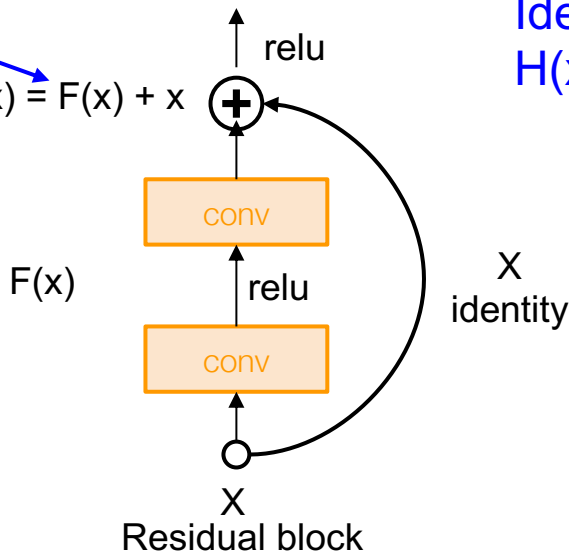
[He et al., 2015]

Solution: Change the network so learning identity functions as extra layers is easy



$$H(x) = F(x) + x$$

$$H(x) = F(x) + x$$



Identity mapping:  
 $H(x) = x$  if  $F(x) = 0$

Use layers to fit **residual**  
 $F(x) = H(x) - x$   
instead of  
 $H(x)$  directly

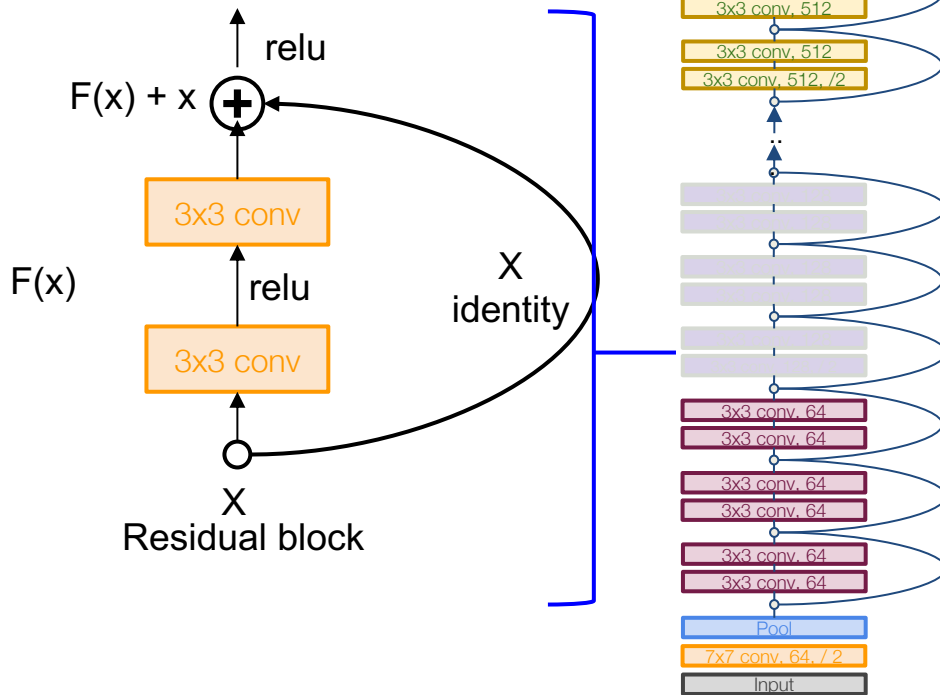


# Case Study: ResNet

[He et al., 2015]

Full ResNet architecture:

- Stack residual blocks
- Every residual block has two 3x3 conv layers

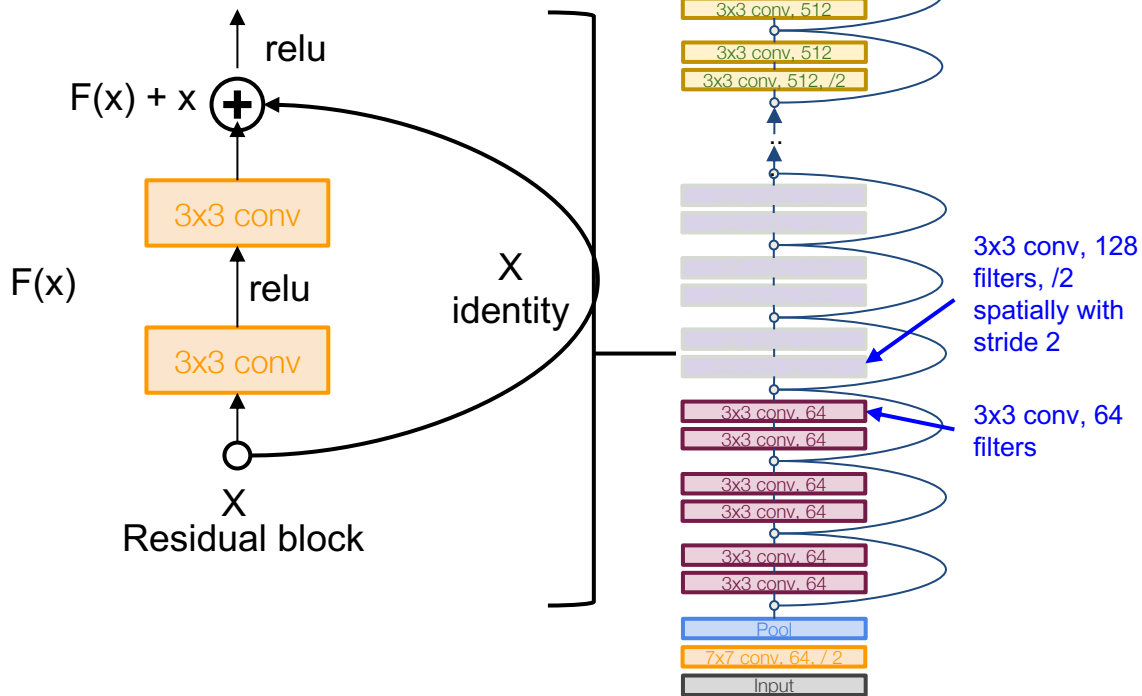


# Case Study: ResNet

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Full ResNet architecture:

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  - Periodically, double # of filters and downsample spatially using stride 2 (/2 in each dimension)
- Reduce the activation volume by half.

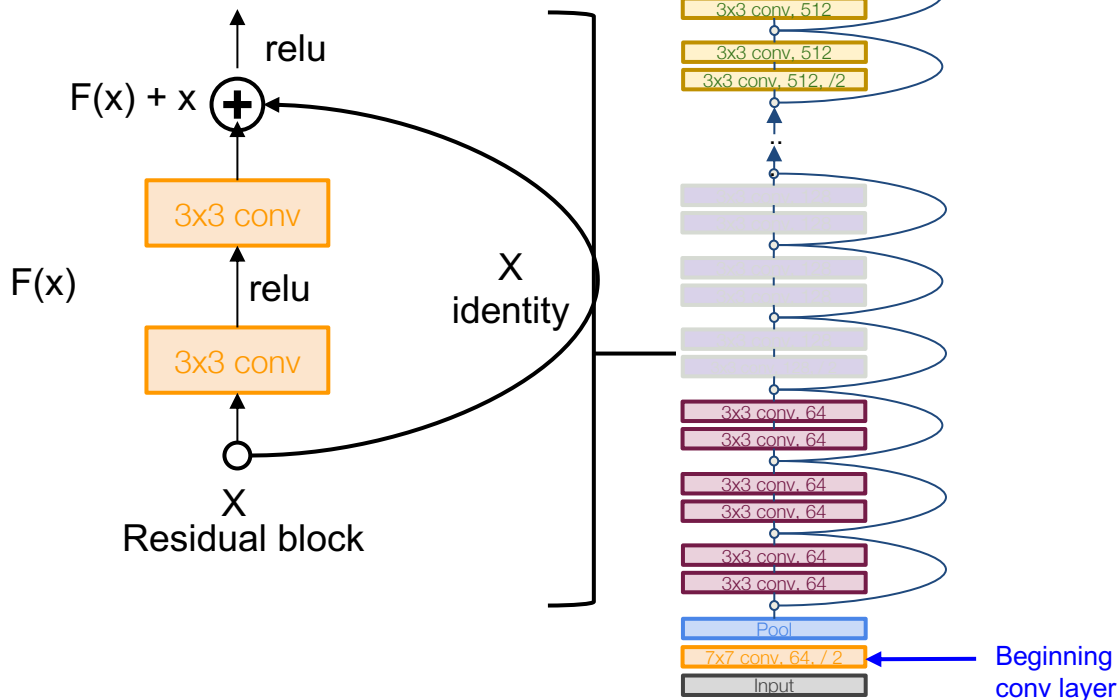


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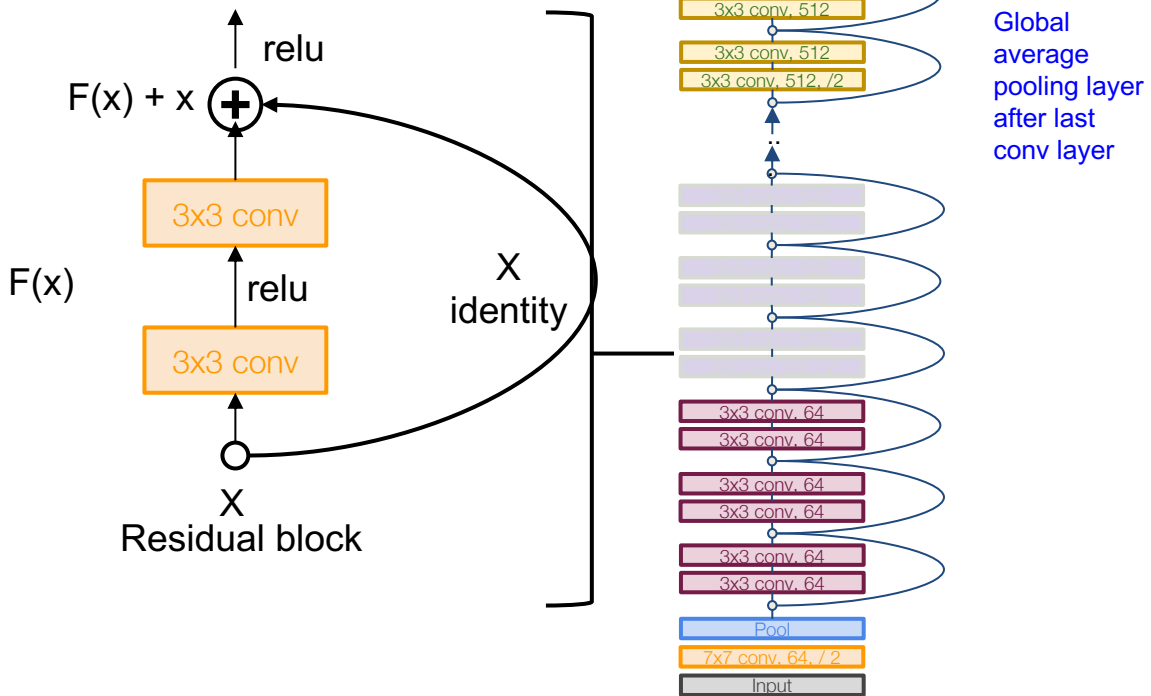


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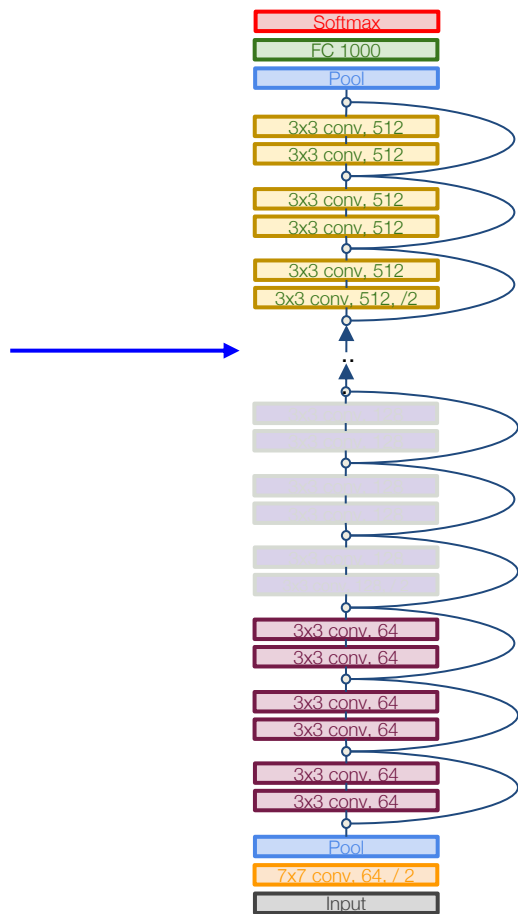
- Stack residual blocks
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- Periodically, double # of filters and downsample spatially using stride 2 (/2 in each dimension)  
Reduce the activation volume by half.
- Additional conv layer at the beginning (stem)
- No FC layers at the end (only FC 1000 to output classes)



# Case Study: ResNet

[He et al., 2015]

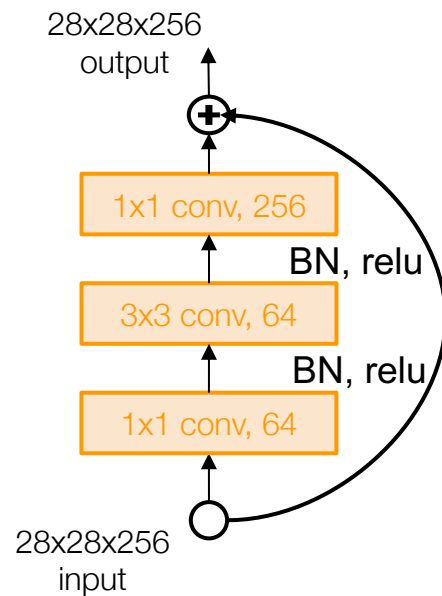
Total depths of 18, 34, 50,  
101, or 152 layers for  
ImageNet



# Case Study: ResNet

[He et al., 2015]

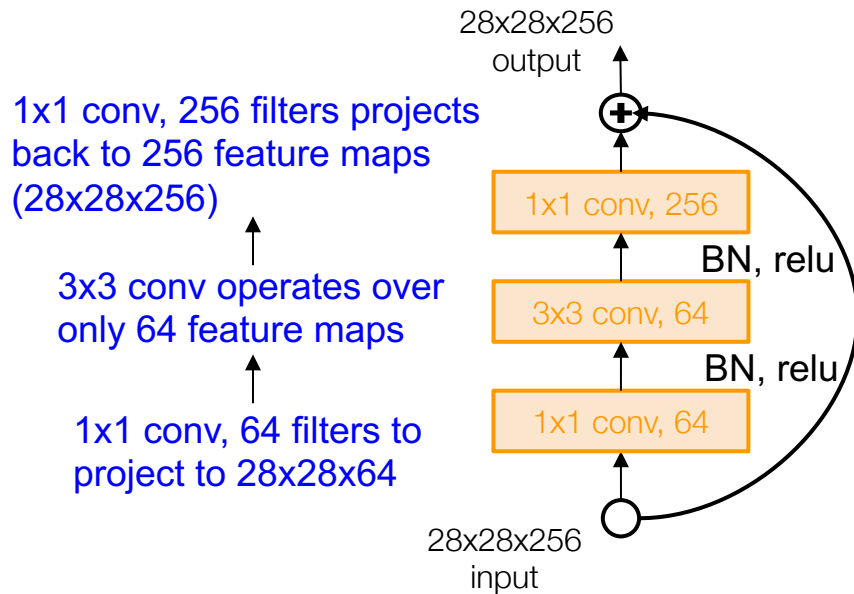
For deeper networks (ResNet-50+), use “bottleneck” layer to improve efficiency (similar to GoogLeNet)



# Case Study: ResNet

[He et al., 2015]

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# Case Study: ResNet

*[He et al., 2015]*

Training ResNet in practice:

- Batch Normalization after every CONV layer (this lecture)
- Xavier initialization from He et al. (this lecture)
- SGD + Momentum (this lecture)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of  $1e-5$
- No dropout used



# Case Study: ResNet

[He et al., 2015]

## Experimental Results

- Able to train very deep networks without degrading (152 layers on ImageNet, 1202 on Cifar)
- Deeper networks now achieve lower training error as expected
- Swept 1st place in all ILSVRC and COCO 2015 competitions

## MSRA @ ILSVRC & COCO 2015 Competitions

- **1st places in all five main tracks**

- ImageNet Classification: *"Ultra-deep"* (quote Yann) **152-layer** nets
- ImageNet Detection: **16%** better than 2nd
- ImageNet Localization: **27%** better than 2nd
- COCO Detection: **11%** better than 2nd
- COCO Segmentation: **12%** better than 2nd

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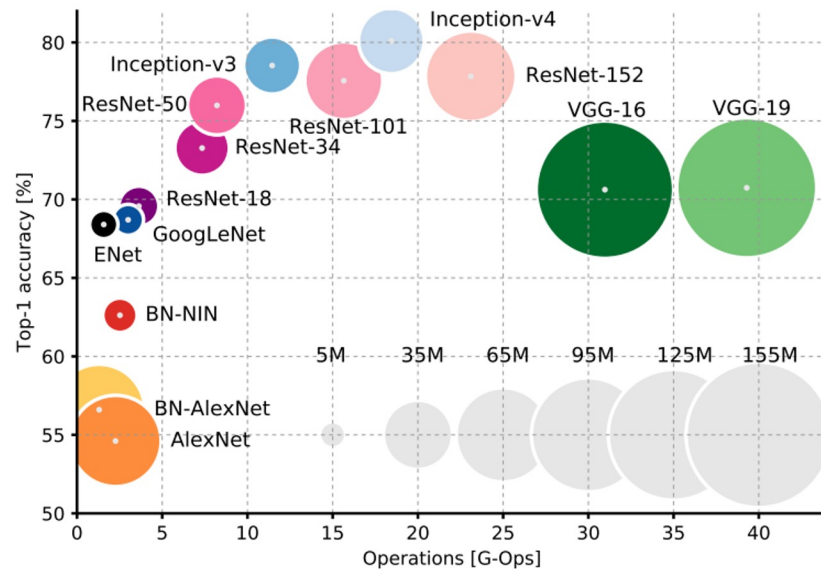
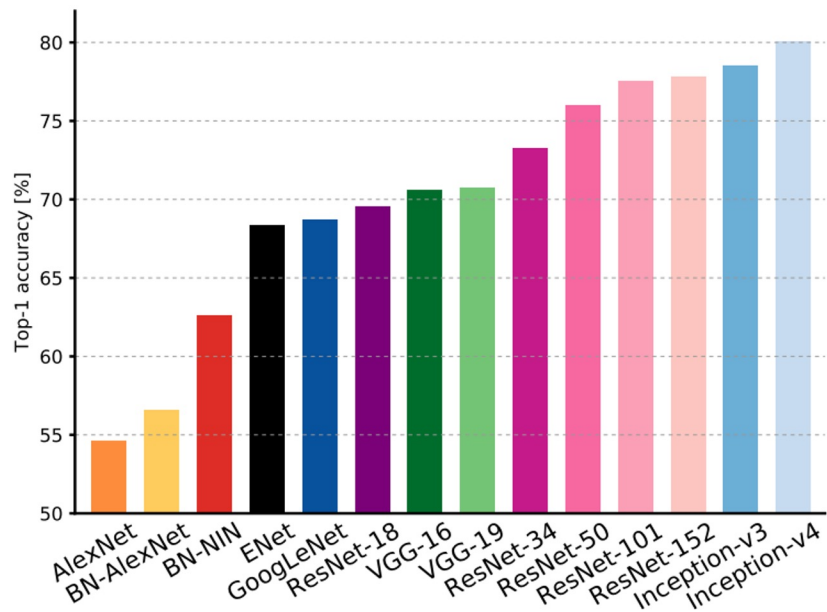
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- COCO Detection: 11% better than 2nd
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ILSVRC 2015 classification winner (3.6% top 5 error) -- better than “human performance”! (Russakovsky 2014)

# Comparing complexity...

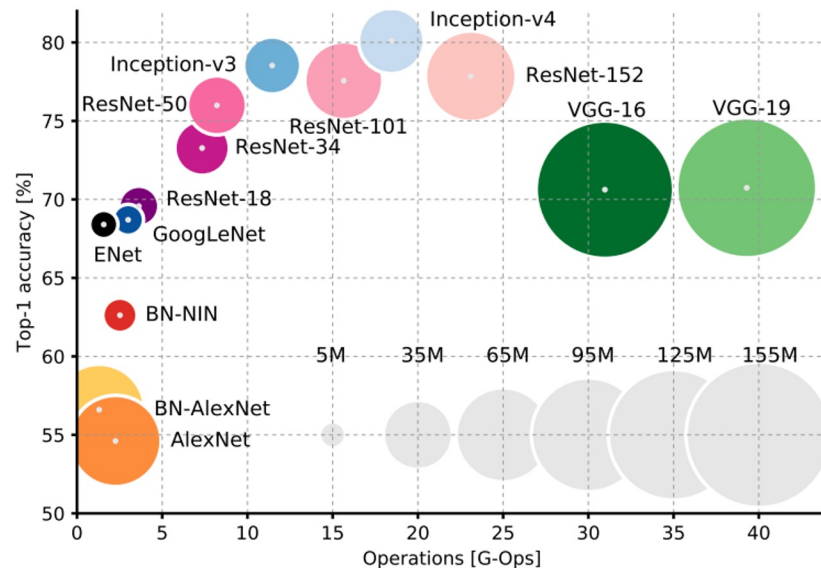
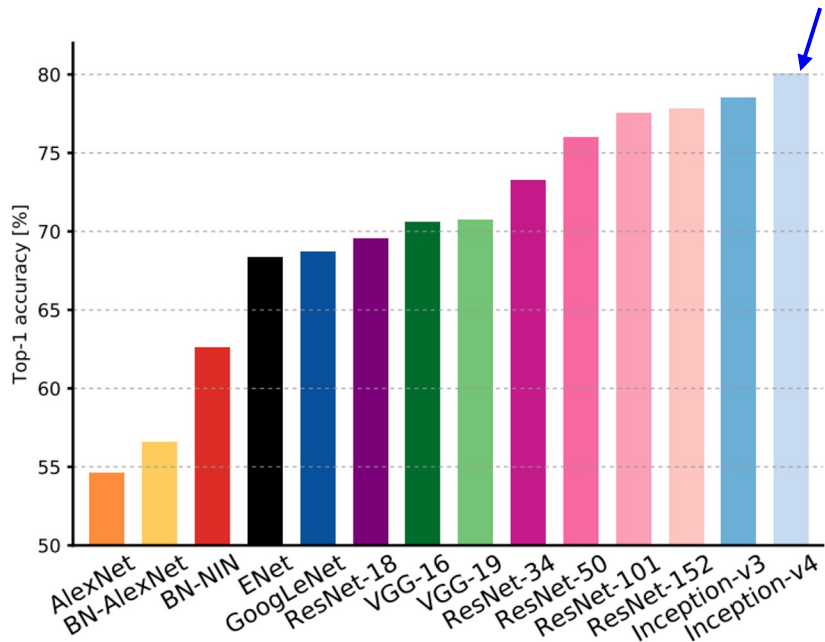


An Analysis of Deep Neural Network Models for Practical Applications, 2017.

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# Comparing complexity...

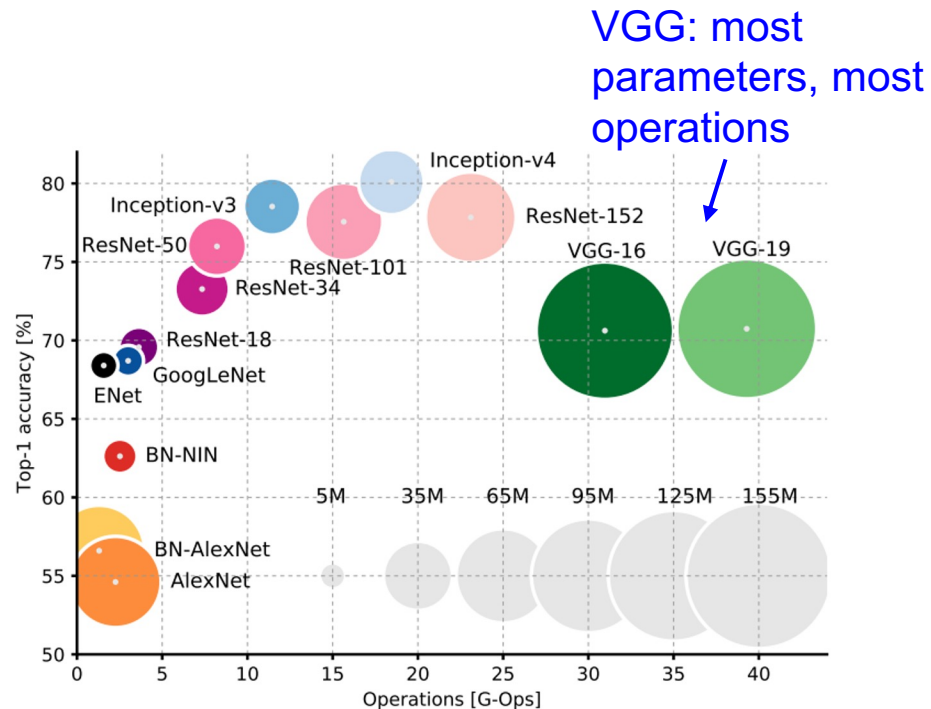
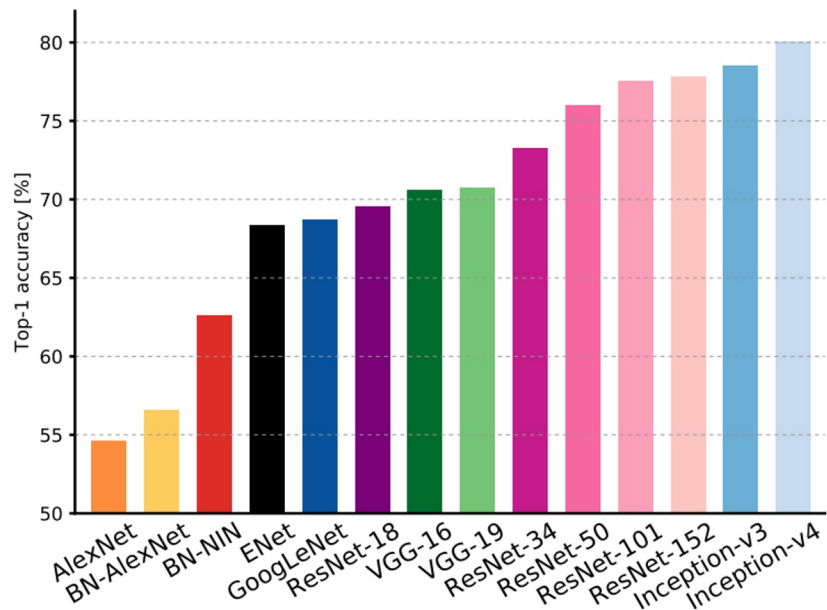
Inception-v4: Resnet + Inception!



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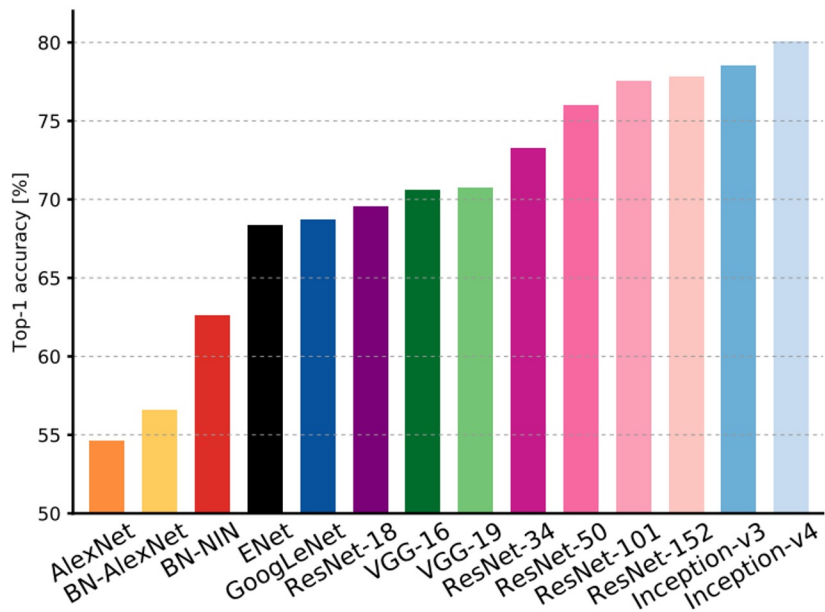
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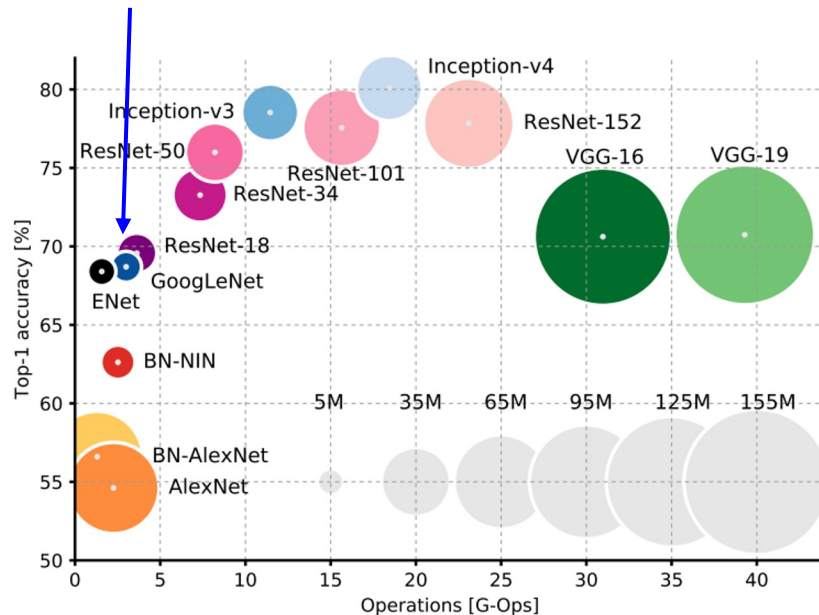
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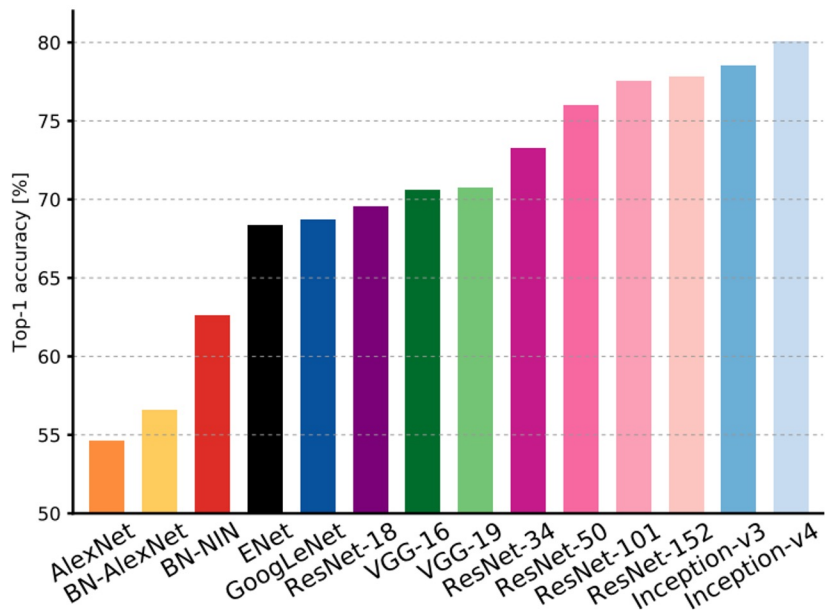
GoogLeNet:  
most efficient



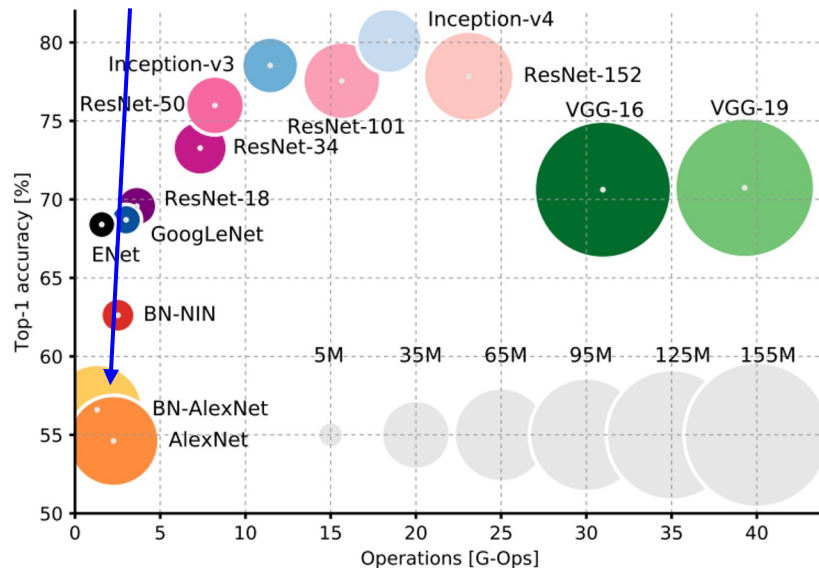
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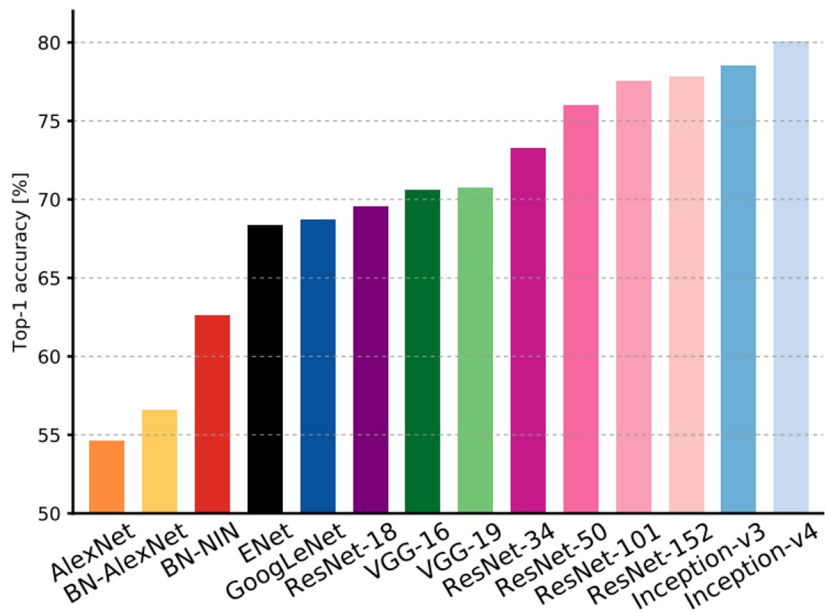
AlexNet:  
Smaller compute, still memory heavy, lower accuracy



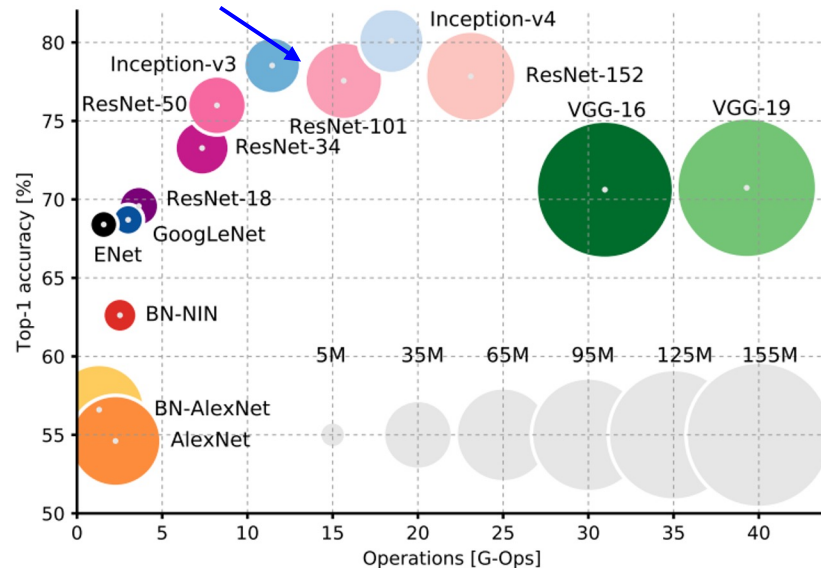
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# Comparing complexity...



ResNet:  
Moderate efficiency depending on model, highest accuracy

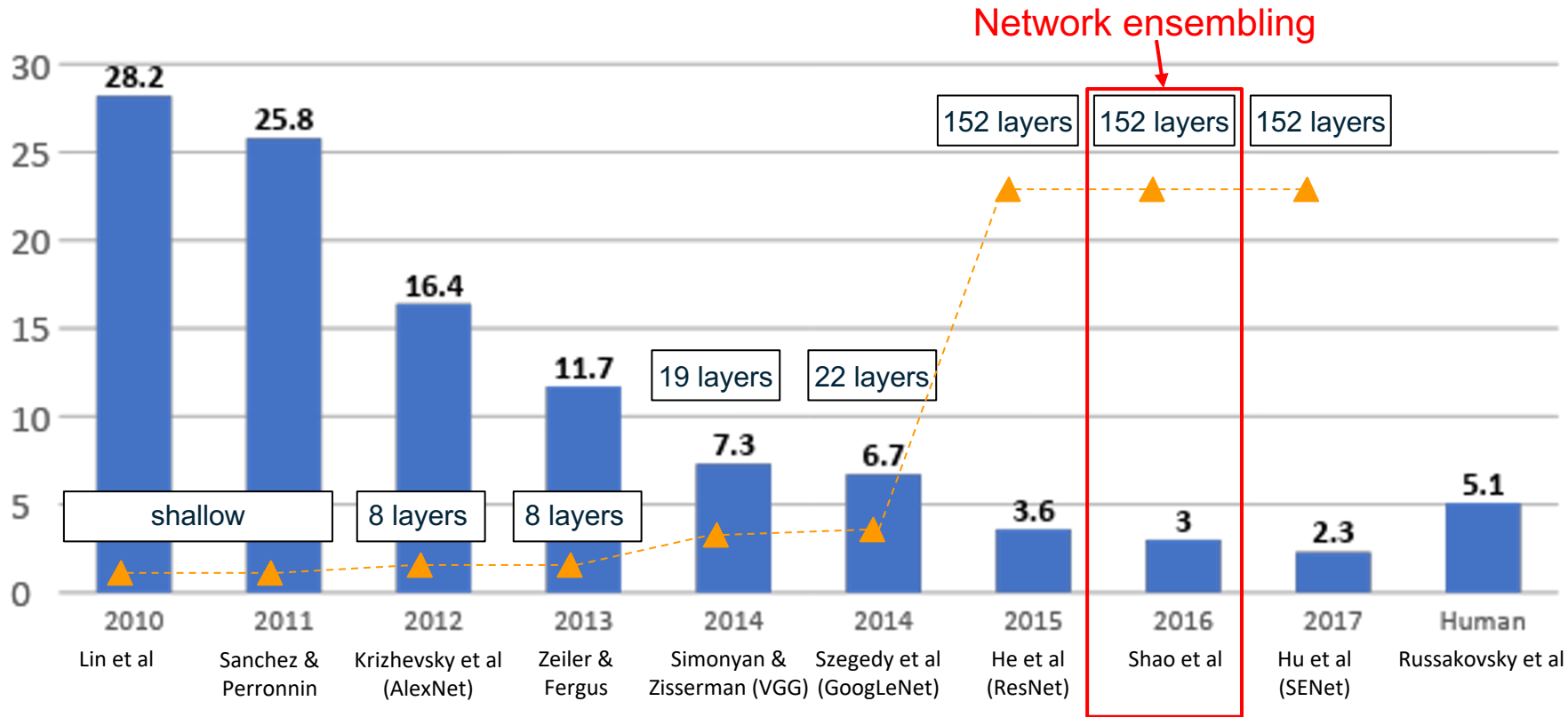


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# ImageNet Large Scale Visual Recognition Challenge (ILSVRC) winners



# Improving ResNets...

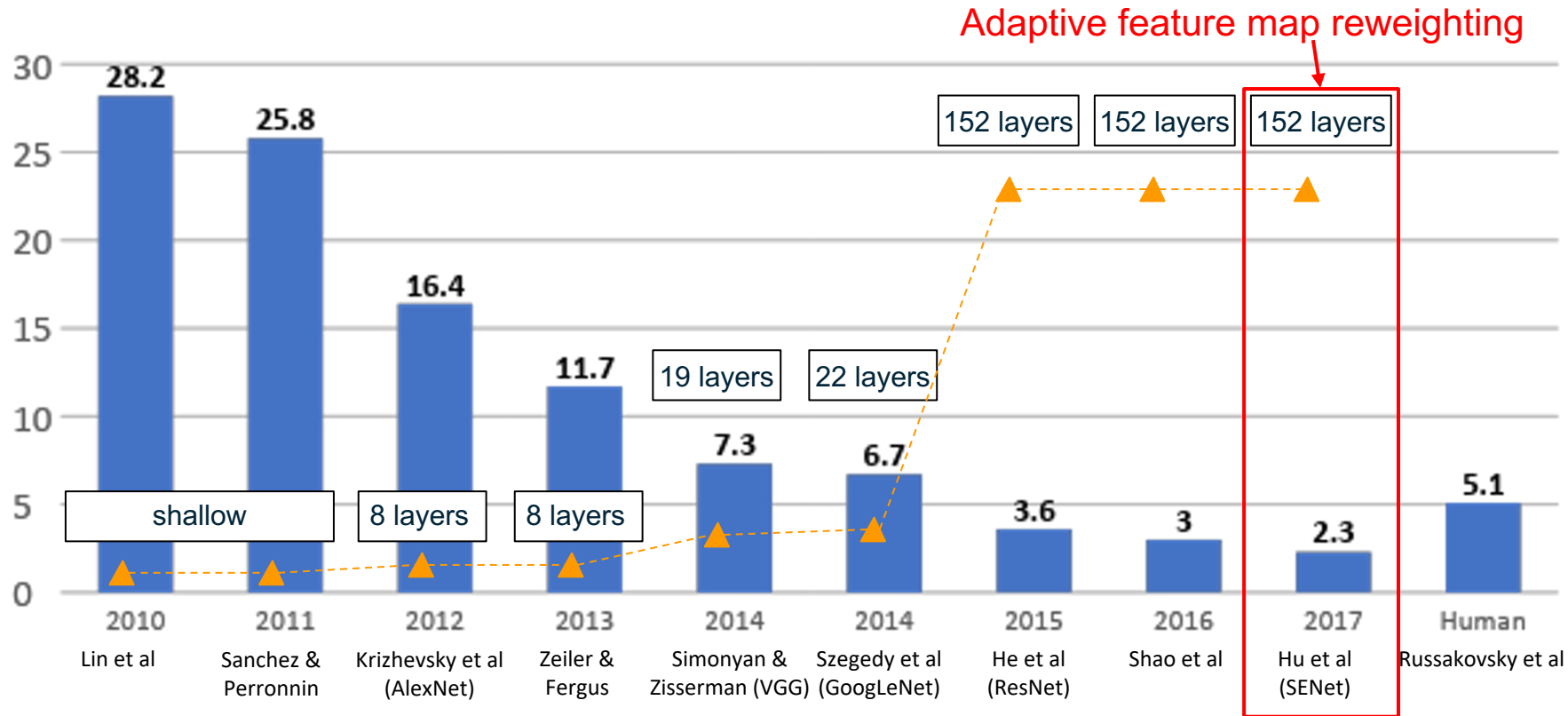
## “Good Practices for Deep Feature Fusion”

[Shao et al. 2016]

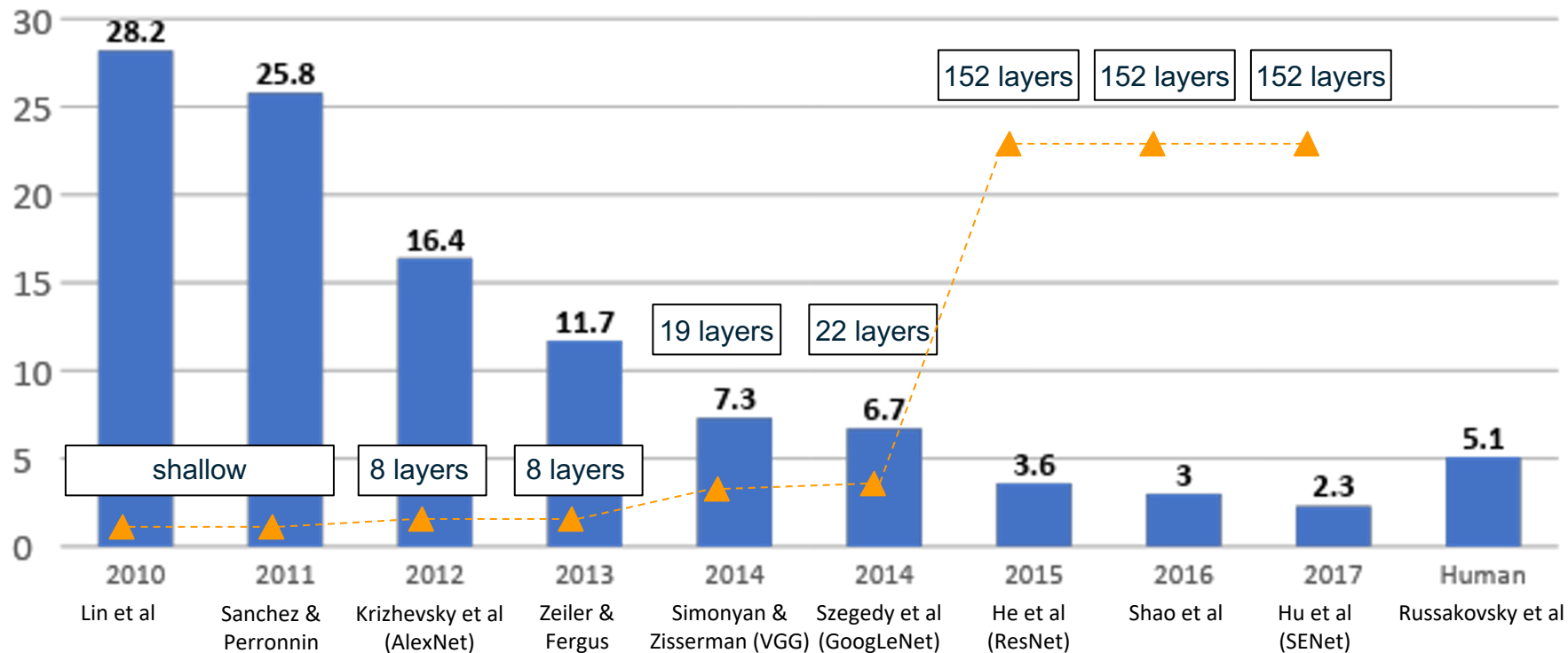
- Multi-scale ensembling of Inception, Inception-Resnet, Resnet, Wide Resnet models
- ILSVRC'16 classification winner

	Inception-v3	Inception-v4	Inception-Resnet-v2	Resnet-200	Wrn-68-3	Fusion (Val.)	Fusion (Test)
Err. (%)	4.20	4.01	3.52	4.26	4.65	2.92 (-0.6)	2.99

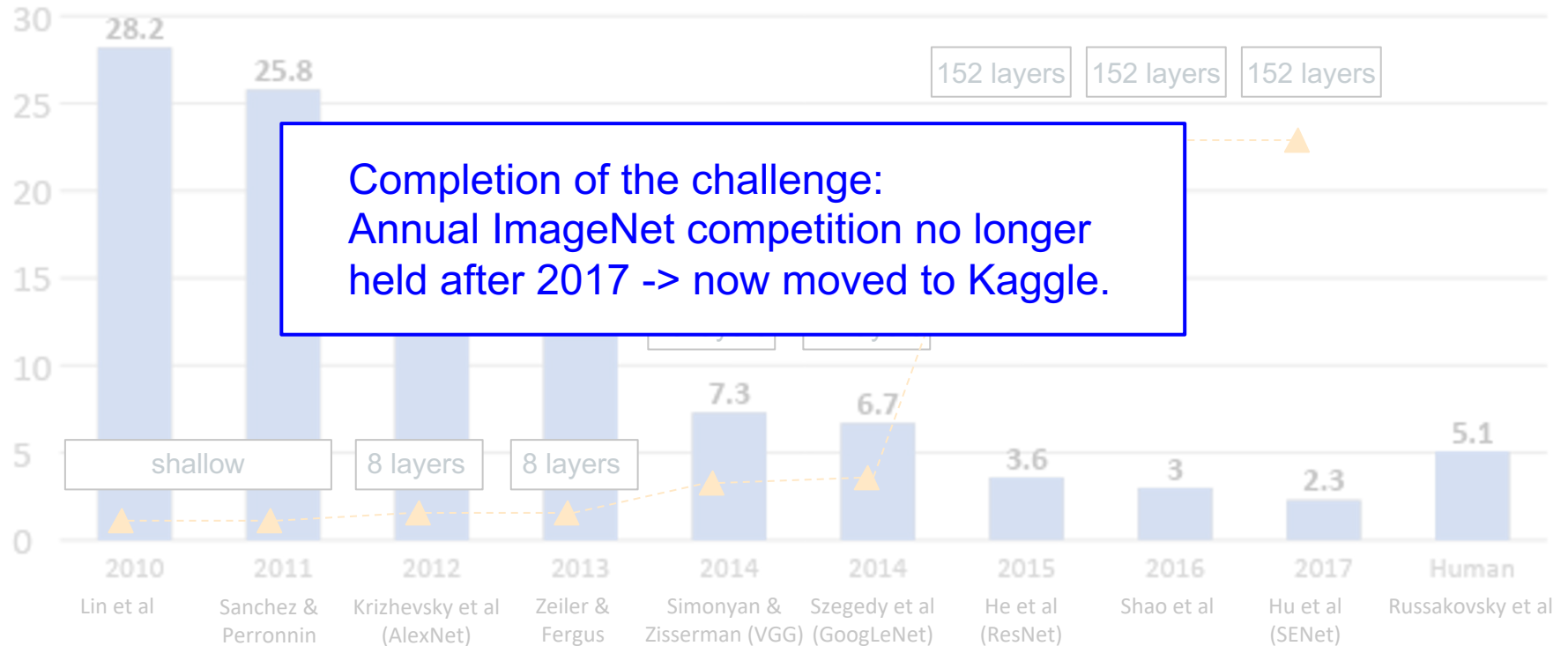
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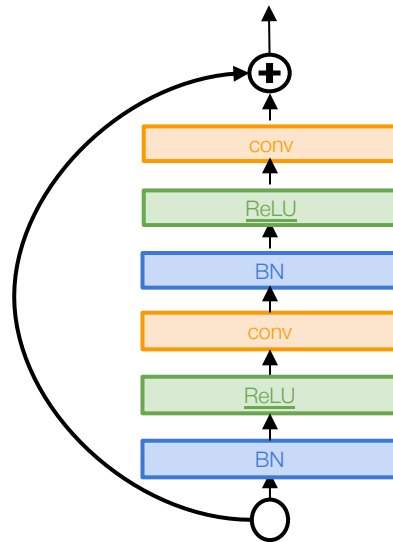
But research into CNN architectures is still flourishing

# Improving ResNets...

## Identity Mappings in Deep Residual Networks

[He et al. 2016]

- Improved ResNet block design from creators of ResNet
- Creates a more direct path for propagating information throughout network
- Gives better performance

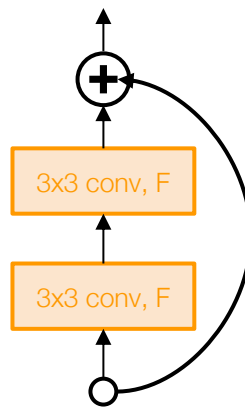


# Improving ResNets...

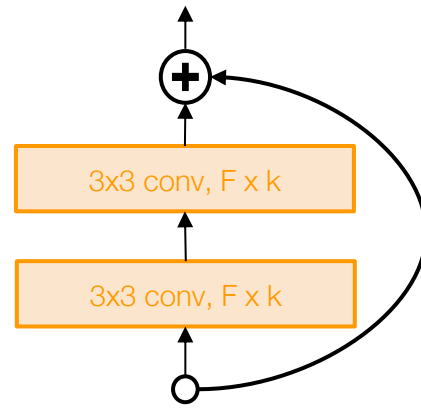
## Wide Residual Networks

[Zagoruyko et al. 2016]

- Argues that residuals are the important factor, not depth
- Use wider residual blocks ( $F \times k$  filters instead of  $F$  filters in each layer)
- 50-layer wide ResNet outperforms 152-layer original ResNet
- Increasing width instead of depth more computationally efficient (parallelizable)



Basic residual block



Wide residual block

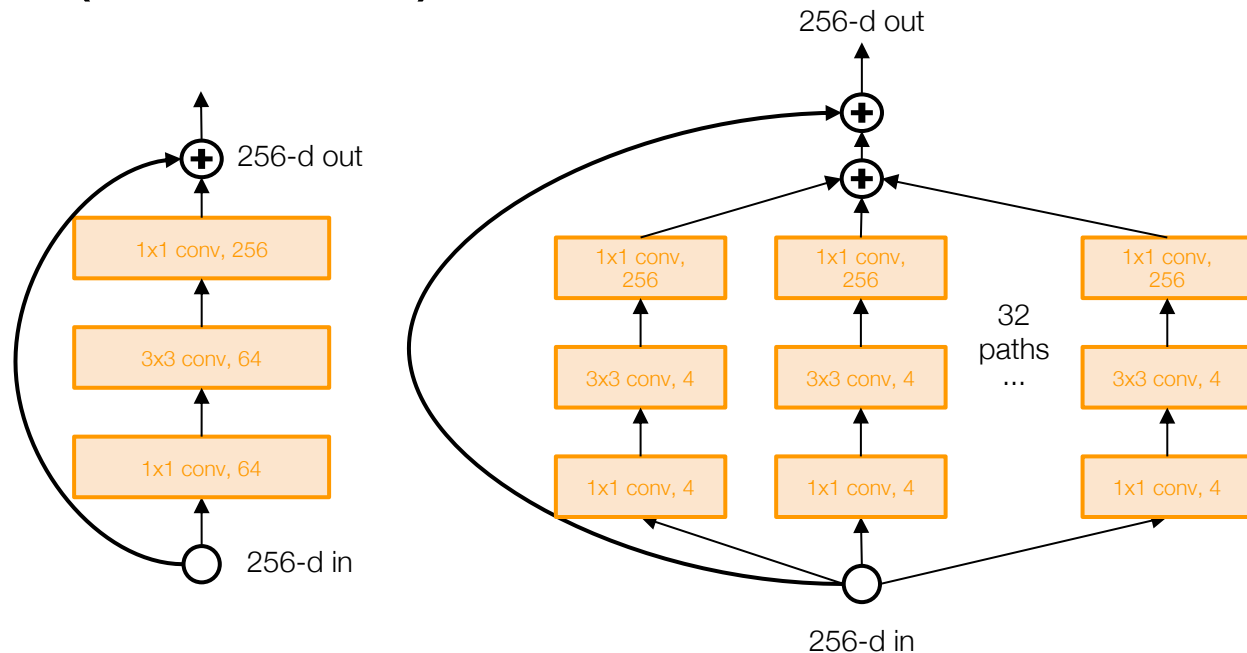


# Improving ResNets...

## Aggregated Residual Transformations for Deep Neural Networks (ResNeXt)

[Xie et al. 2016]

- Also from creators of ResNet
- Increases width of residual block through multiple parallel pathways (“cardinality”)
- Parallel pathways similar in spirit to Inception module

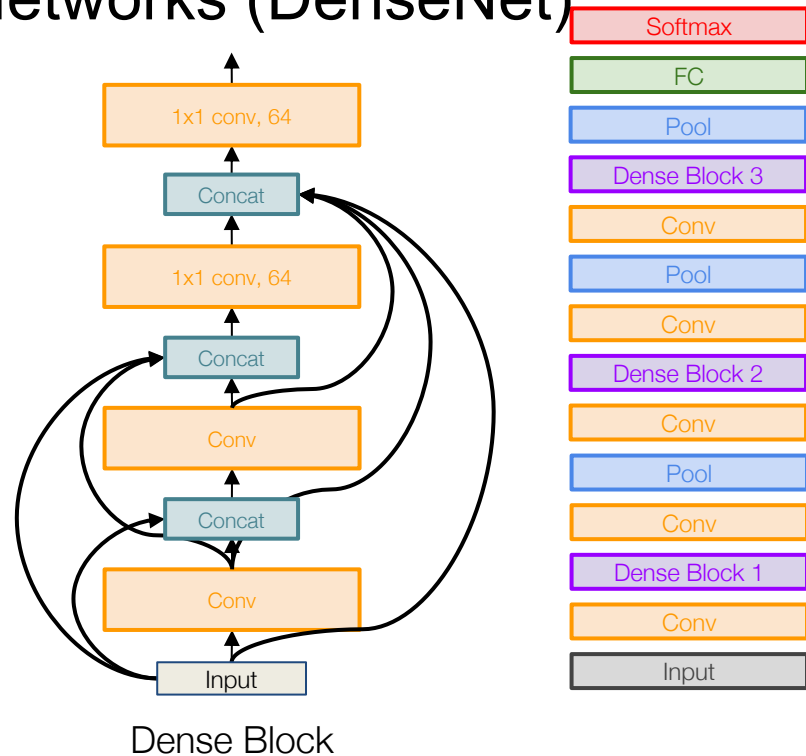


# Other ideas...

## Densely Connected Convolutional Networks (DenseNet)

[Huang et al. 2017]

- Dense blocks where each layer is connected to every other layer in feedforward fashion
- Alleviates vanishing gradient, strengthens feature propagation, encourages feature reuse
- Showed that shallow 50-layer network can outperform deeper 152 layer ResNet

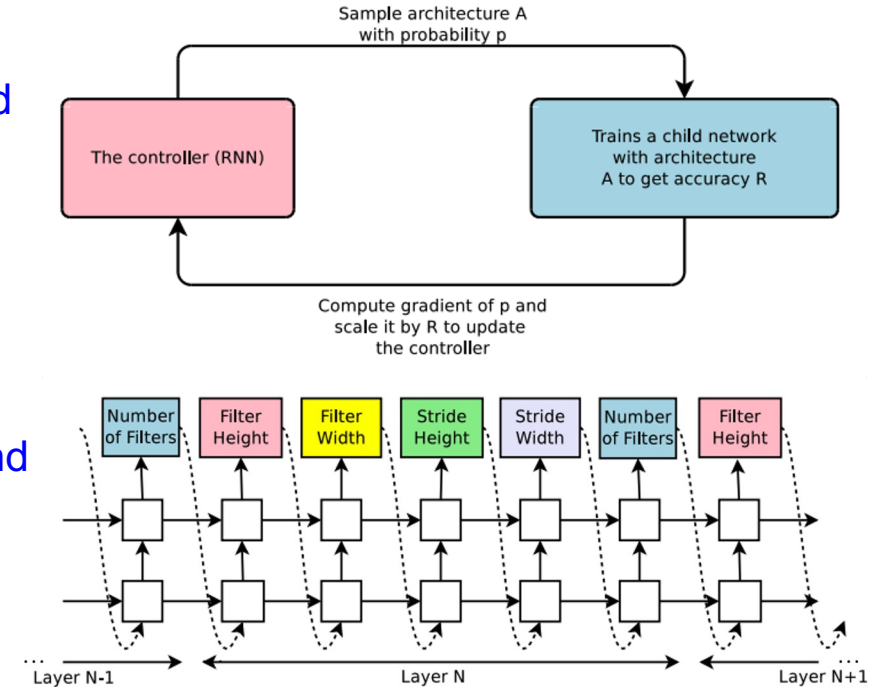


# Learning to search for network architectures...

## Neural Architecture Search with Reinforcement Learning (NAS)

[Zoph et al. 2016]

- “Controller” network that learns to design a good network architecture (output a string corresponding to network design)
- Iterate:
  - 1) Sample an architecture from search space
  - 2) Train the architecture to get a “reward”  $R$  corresponding to accuracy
  - 3) Compute gradient of sample probability, and scale by  $R$  to perform controller parameter update (i.e. increase likelihood of good architecture being sampled, decrease likelihood of bad architecture)

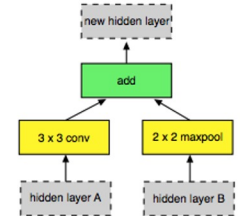
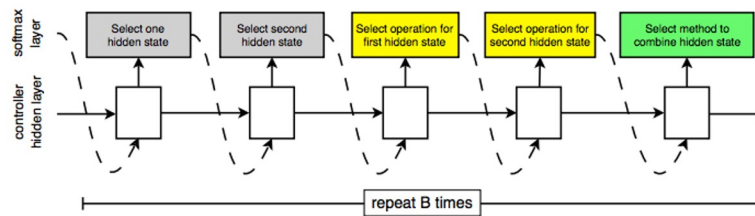
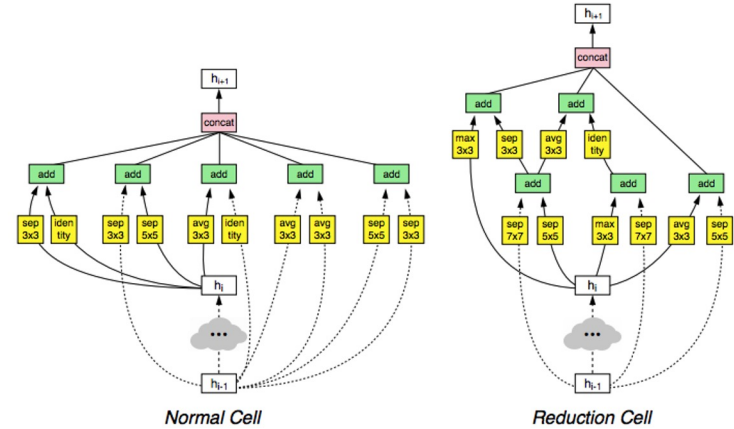


# Learning to search for network architectures...

## Learning Transferable Architectures for Scalable Image Recognition

[Zoph et al. 2017]

- Applying neural architecture search (NAS) to a large dataset like ImageNet is expensive
- Design a search space of building blocks (“cells”) that can be flexibly stacked
- NASNet: Use NAS to find best cell structure on smaller CIFAR-10 dataset, then transfer architecture to ImageNet
- Many follow-up works in this space e.g. AmoebaNet (Real et al. 2019) and ENAS (Pham, Guan et al. 2018)



# But sometimes smart heuristic is better than NAS ...

## EfficientNet: Smart Compound Scaling

[Tan and Le. 2019]

- Increase network capacity by scaling width, depth, and resolution, while balancing accuracy and efficiency.
- Search for optimal set of compound scaling factors given a compute budget (target memory & flops).
- Scale up using smart heuristic rules

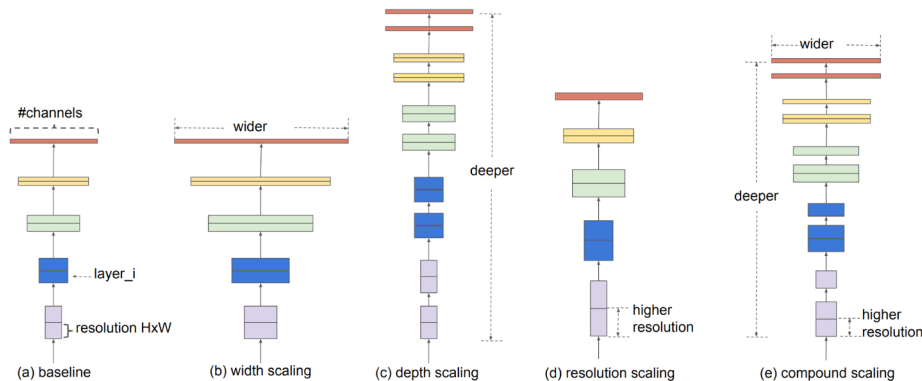
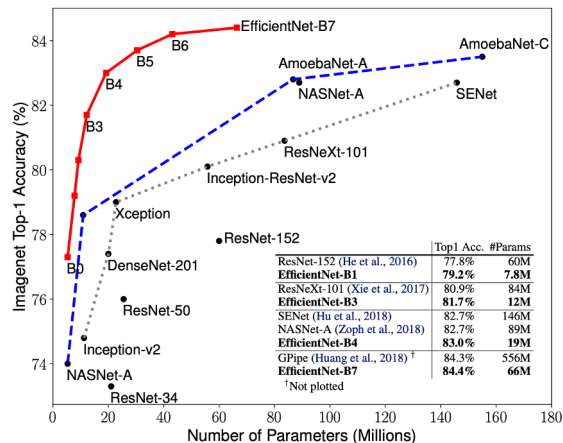
$$\text{depth: } d = \alpha^\phi$$

$$\text{width: } w = \beta^\phi$$

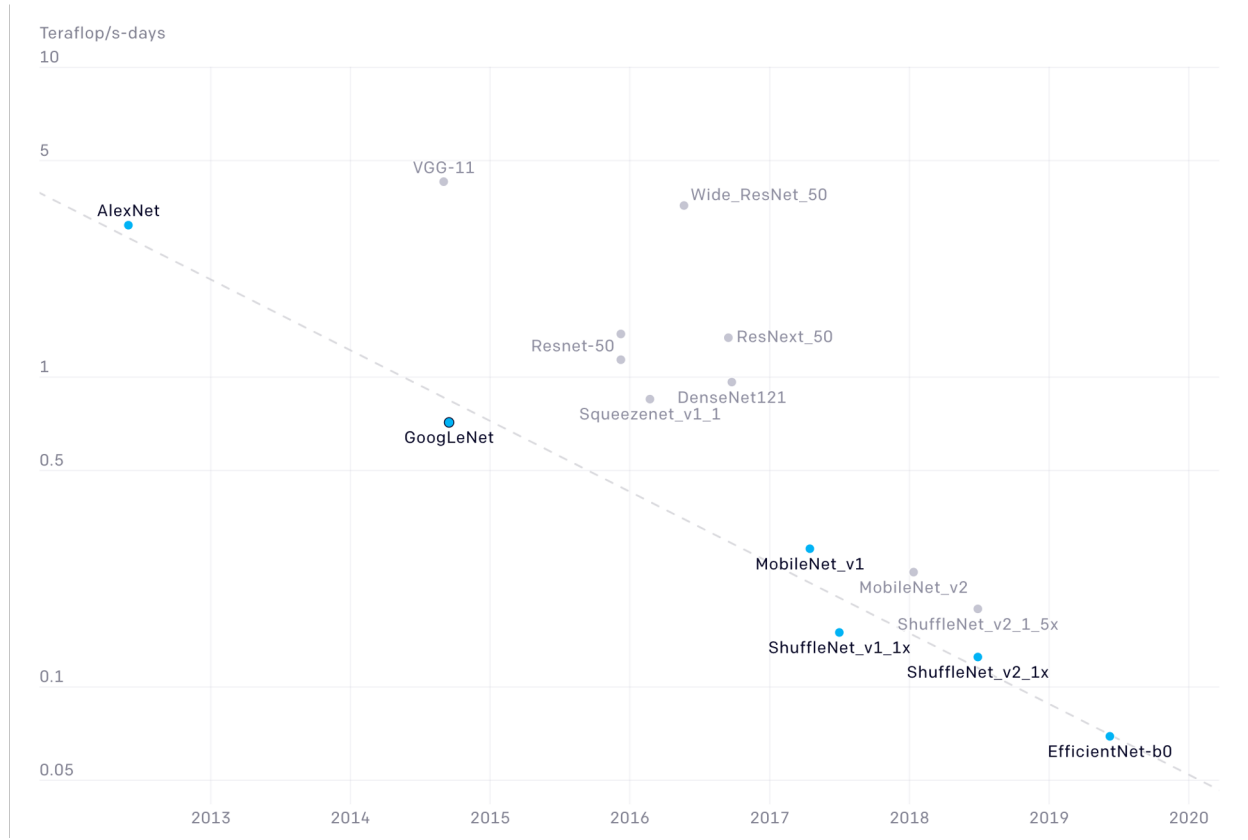
$$\text{resolution: } r = \gamma^\phi$$

$$\text{s.t. } \alpha \cdot \beta^2 \cdot \gamma^2 \approx 2$$

$$\alpha \geq 1, \beta \geq 1, \gamma \geq 1$$



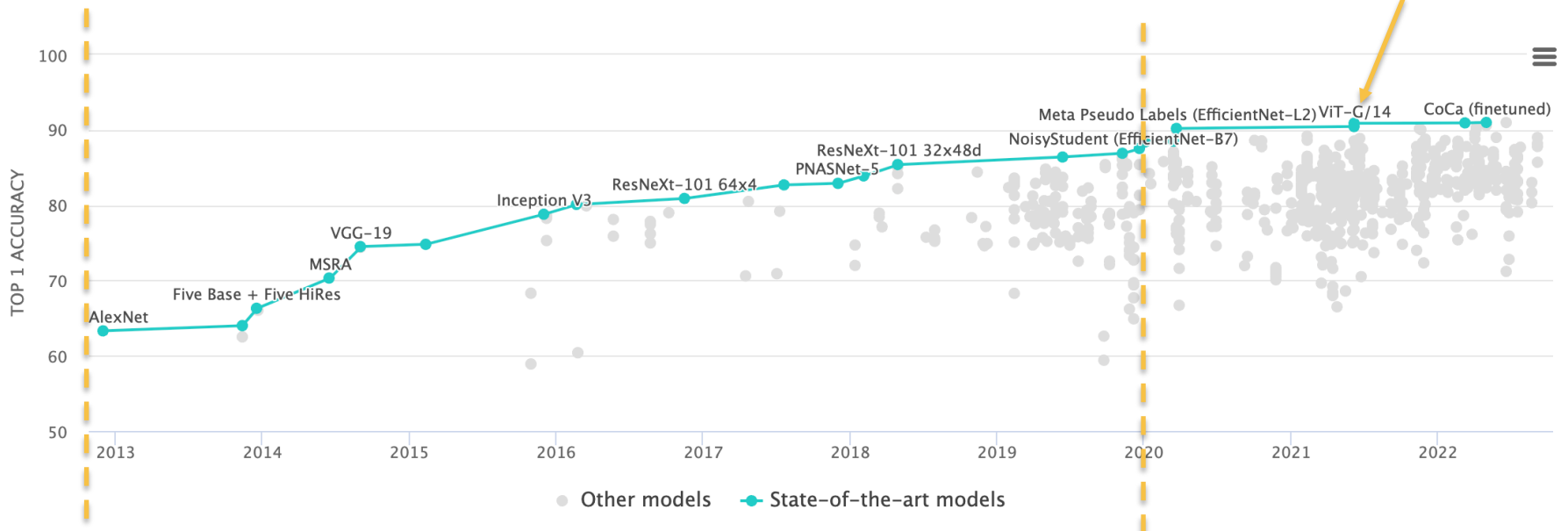
# Amount of compute required to reach “AlexNet performance”



<https://openai.com/blog/ai-and-efficiency/>

This Lecture

Transformer  
(later this sem.)



<https://paperswithcode.com/sota/image-classification-on-imagenet>

# What we have learned so far ...

## Deep Neural Networks:

- What they are (composite parametric, non-linear functions)
- Where they come from (biological inspiration, brief history of ANN)
- How they are optimized, in principle (analytical gradient via computational graphs, backpropagation)
- What they look like in practice (Deep ConvNets for vision)



# Next few lectures:

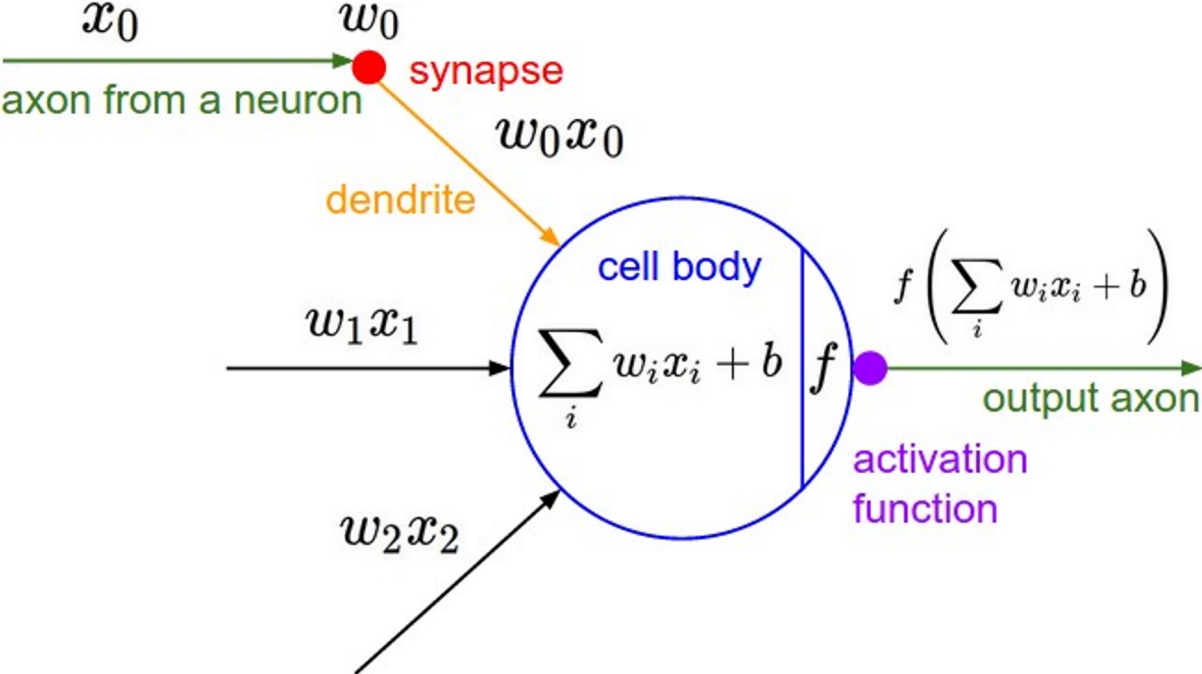
## **Training** Deep Neural Networks

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization
- Batch Normalization
- Regularization
- Advanced Optimization
- Data Augmentation
- Transfer learning
- Hyperparameter Tuning
- Model Ensemble

# Today: Training Deep NNs (Part 1)

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization

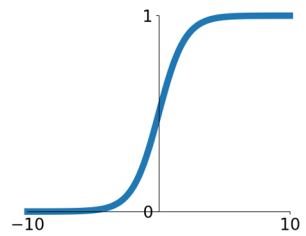
# Activation Functions



# Activation Functions

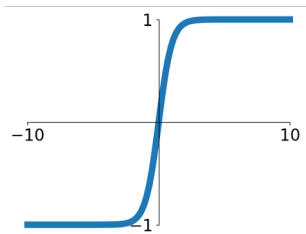
## Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



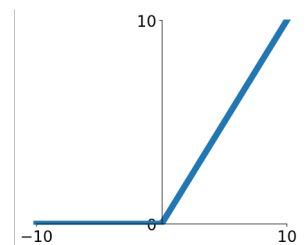
## tanh

$$\tanh(x)$$



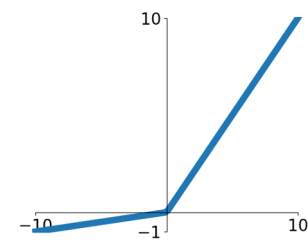
## ReLU

$$\max(0, x)$$



## Leaky ReLU

$$\max(0.1x, x)$$

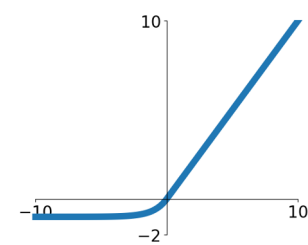


## Maxout

$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

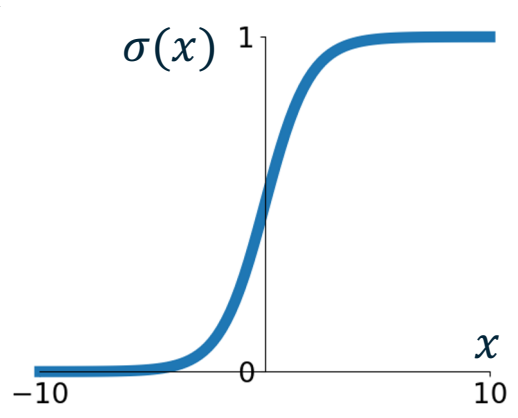
## ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



# Activation Functions

$$\sigma(x) = 1 / (1 + e^{-x})$$

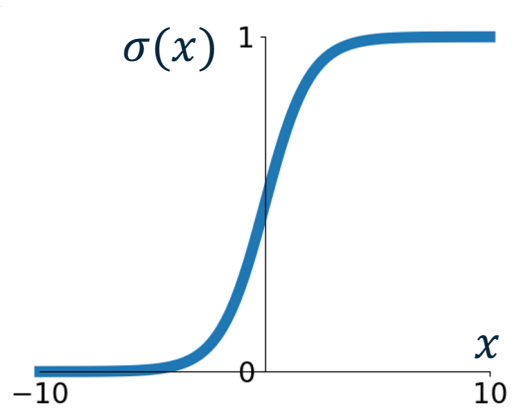


**Sigmoid**

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron

# Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

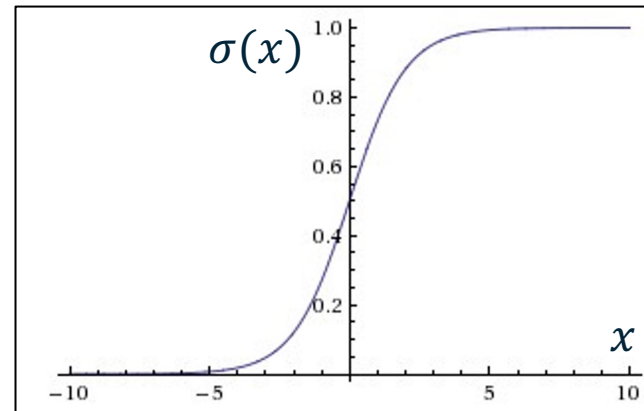
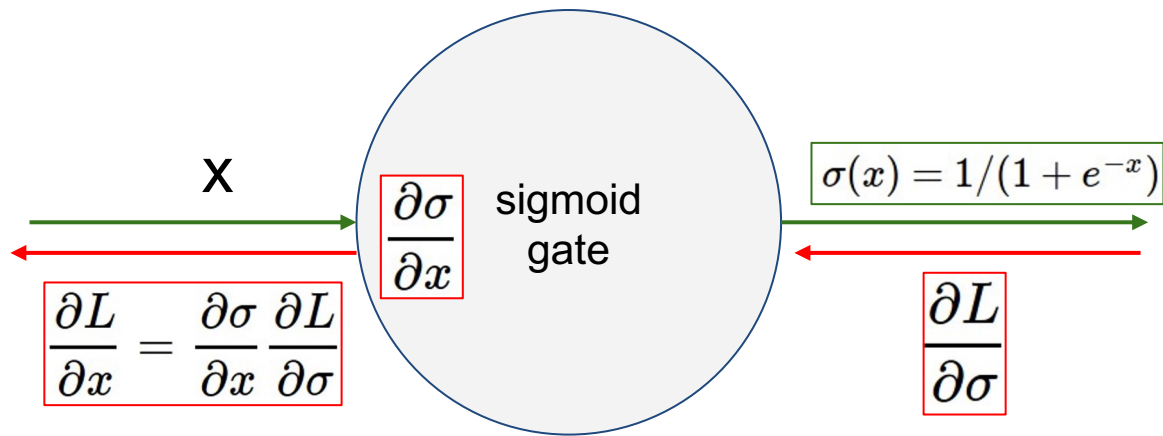


**Sigmoid**

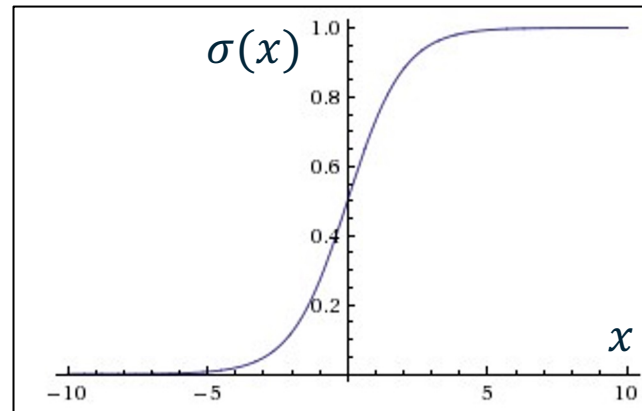
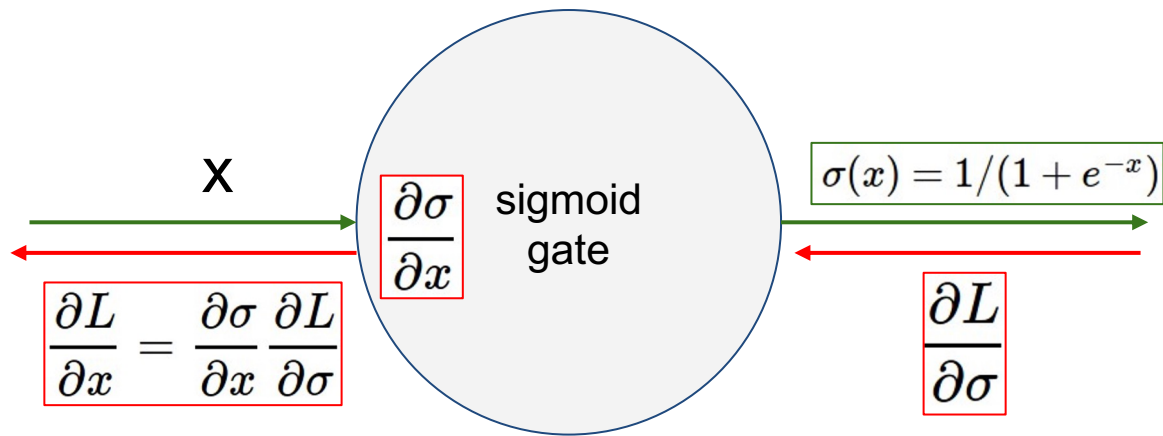
- Squashes numbers to range [0,1]
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Problems:

1. Saturated neurons “kill” the gradients



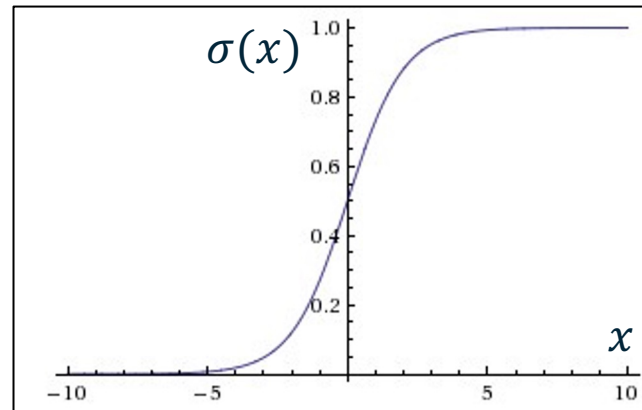
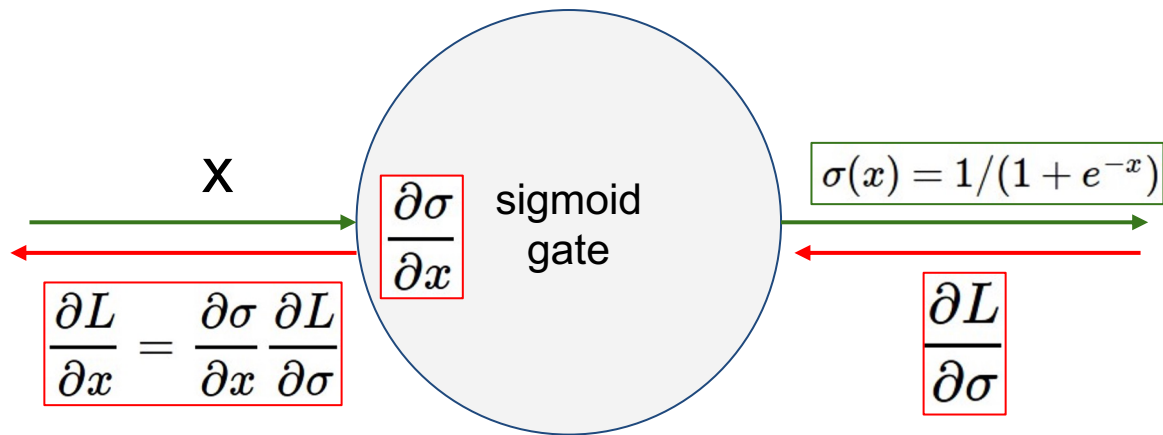
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$



What happens when  $x = -10$ ?

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$



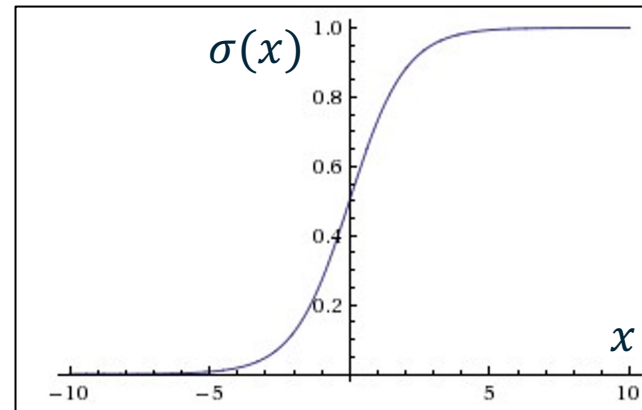
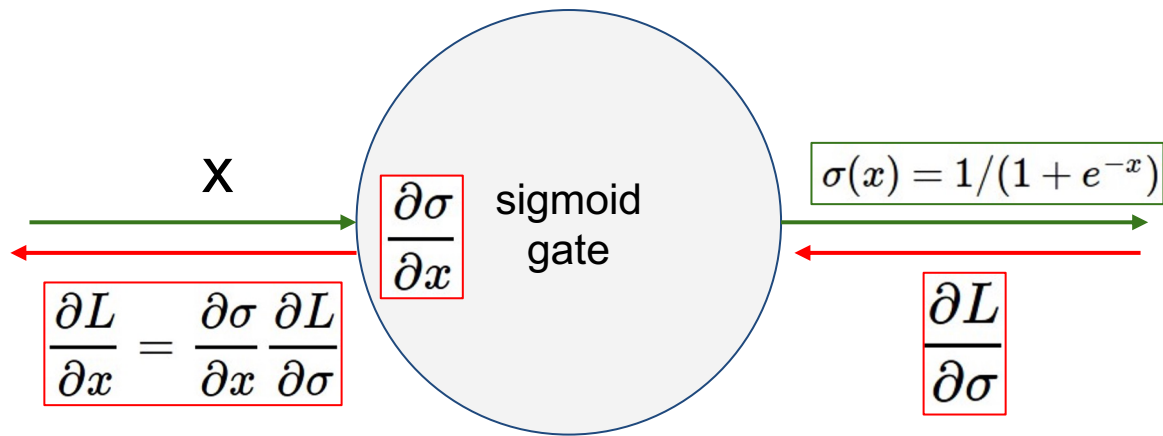


What happens when  $x = -10$ ?

$$\sigma(x) \approx 0$$

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x)) = 0(1 - 0) = 0$$

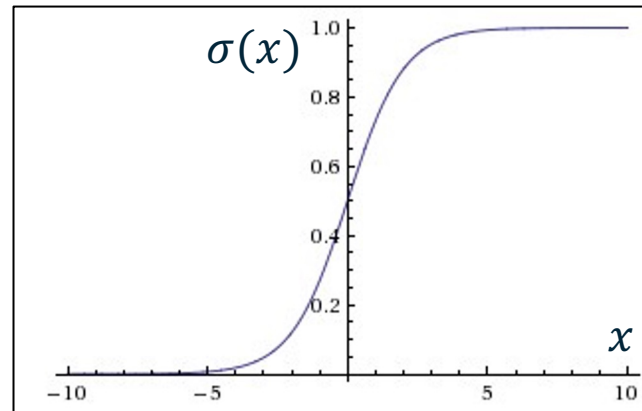
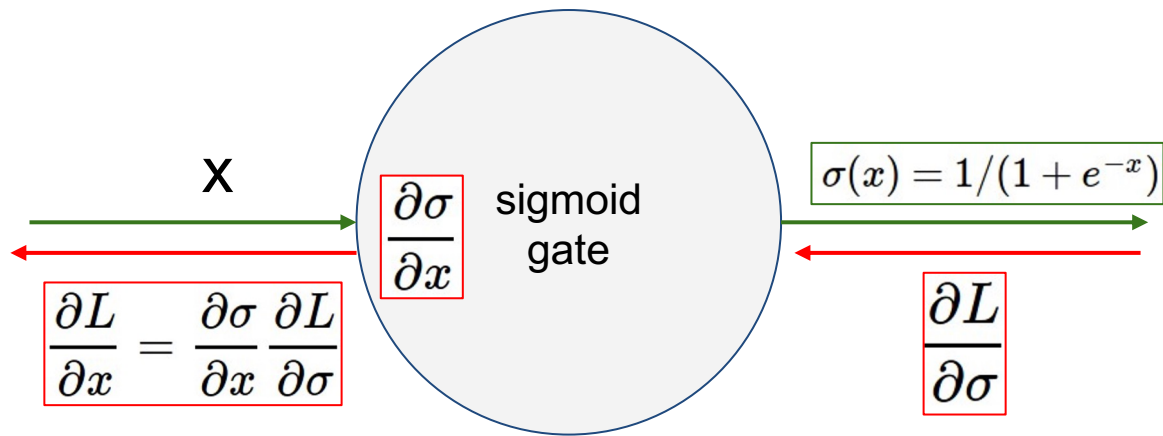
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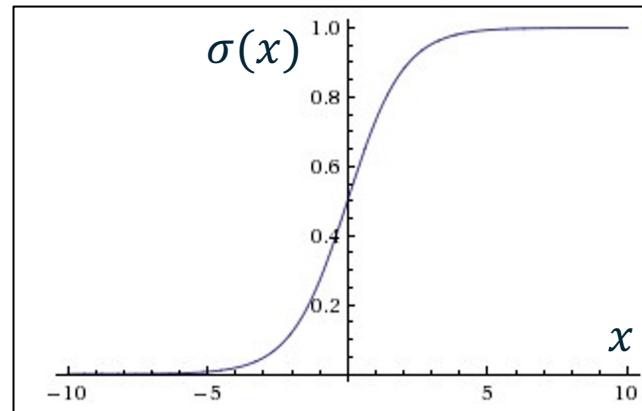
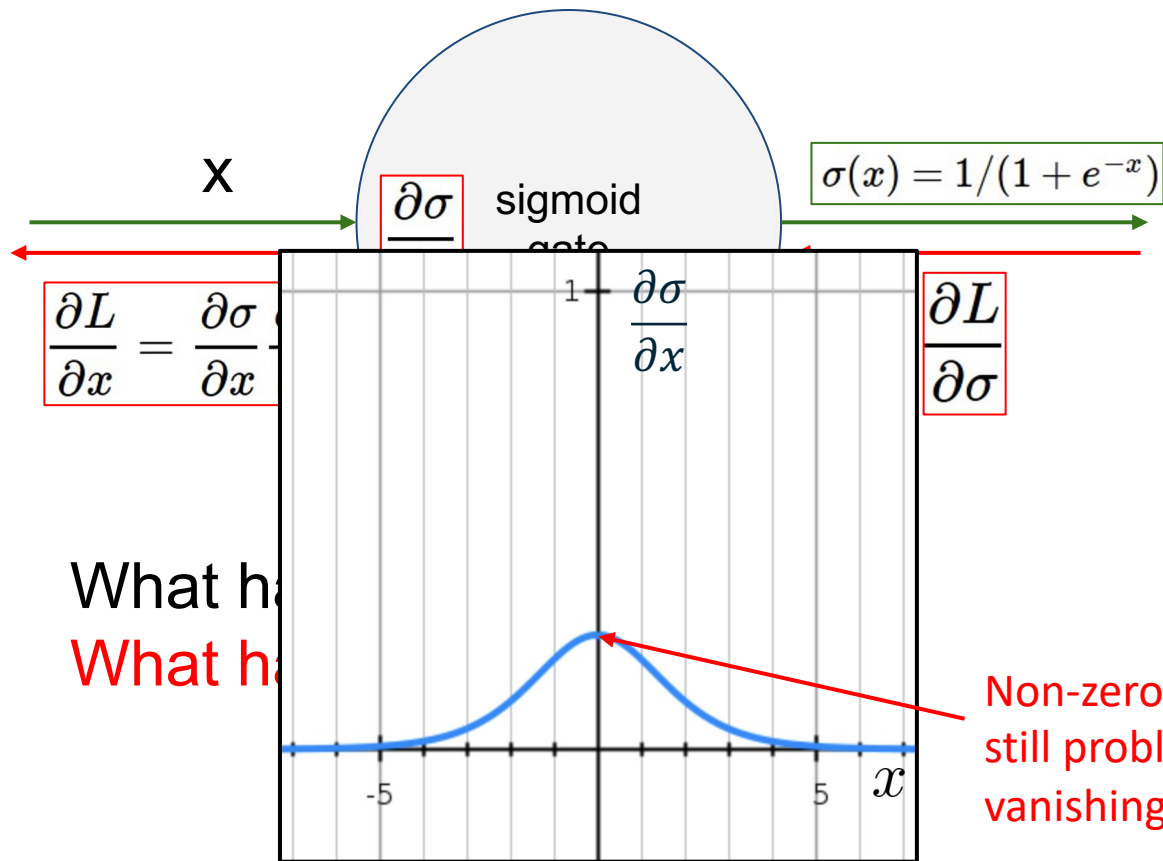


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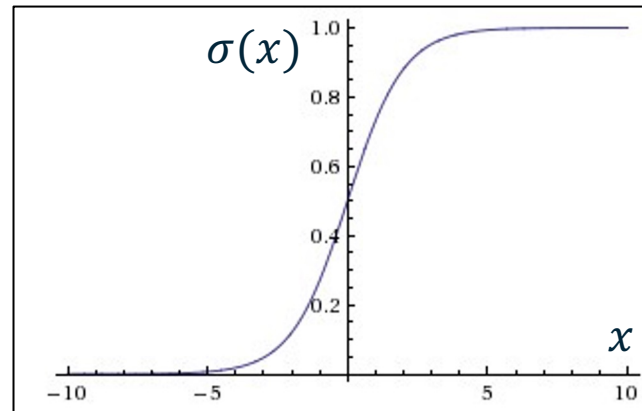
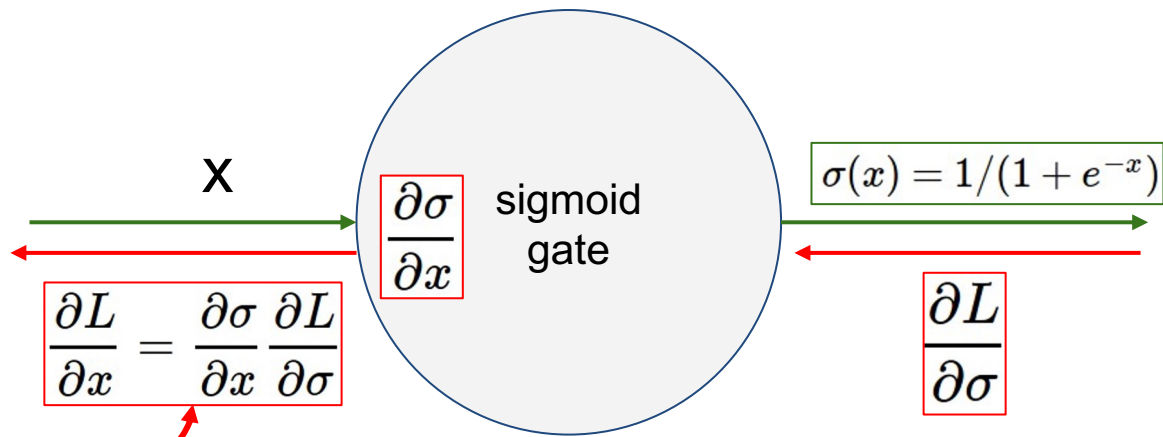
$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

$$\sigma(x) \approx 1 \quad \frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x)) = 1(1 - 1) = 0$$



$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

Non-zero but small:  
 still problematic, causes  
 vanishing gradient



Why is this a problem?

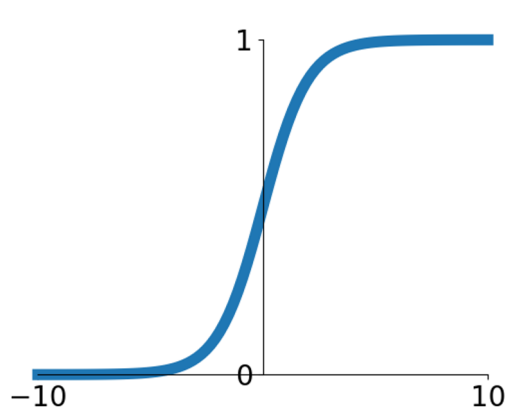
If all the gradients flowing back will be zero and weights will never change (aka “Vanishing Gradient”)

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x) (1 - \sigma(x))$$

# Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron



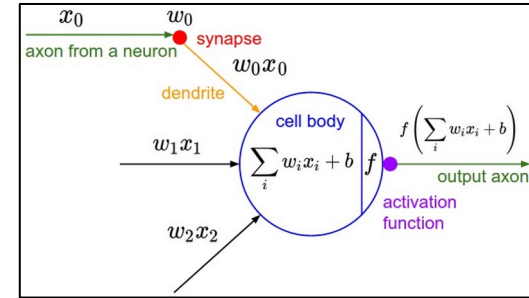
**Sigmoid**

Problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered

Consider what happens when the input to a neuron **is always positive...**

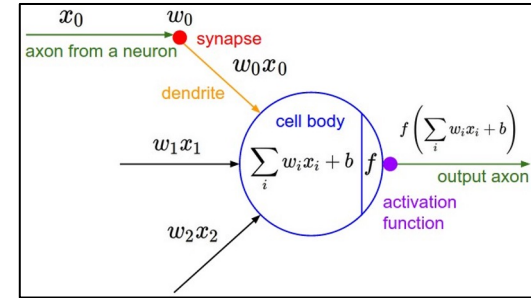
$$f \left( \sum_i w_i x_i + b \right)$$



What can we say about the gradients on  $\mathbf{w}$ ?

Consider what happens when the input to a neuron **is always positive...**

$$f\left(\sum_i w_i x_i + b\right)$$



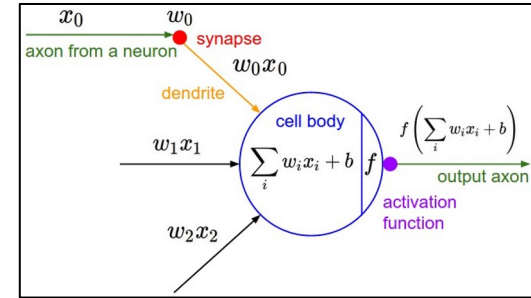
What can we say about the gradients on  $\mathbf{w}$ ?

$$\frac{\partial L}{\partial w} = \sigma\left(\sum_i w_i x_i + b\right)\left(1 - \sigma\left(\sum_i w_i x_i + b\right)\right)x \times \textit{upstream\_gradient}$$



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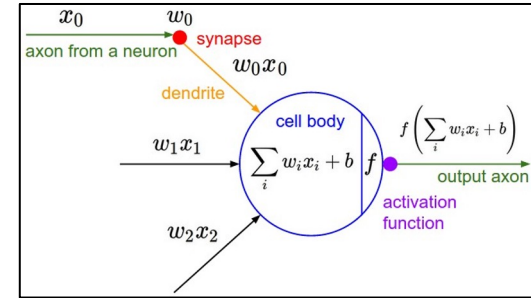
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We know that local gradient of sigmoid is always positive

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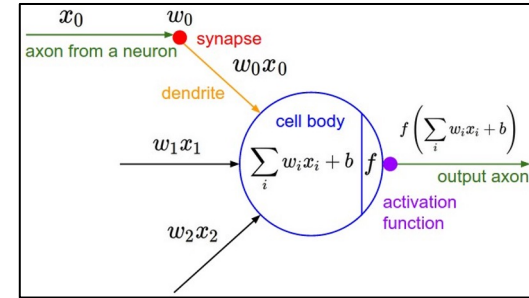
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We are assuming  $x$  is positive

$$\frac{\partial L}{\partial w} = \sigma\left(\sum_i w_i x_i + b\right) \left(1 - \sigma\left(\sum_i w_i x_i + b\right)\right) x \times upstream\_gradient$$

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$



What can we say about the gradients on  $\mathbf{w}$ ?

We know that local gradient of sigmoid is always positive

We are assuming  $x$  is positive

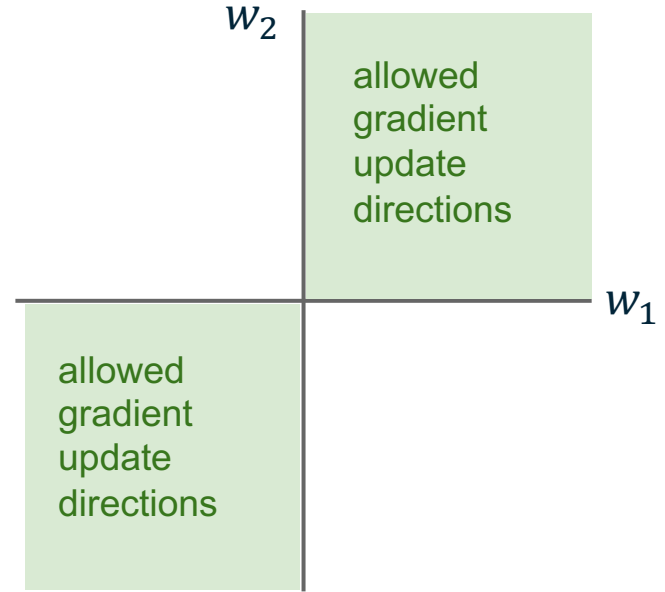
Sign of gradient for all  $w_i$  is the same as the sign of upstream gradient.

That is, local gradient cannot change the sign of global gradient

$$\frac{\partial L}{\partial w} = \sigma\left(\sum_i w_i x_i + b\right) \left(1 - \sigma\left(\sum_i w_i x_i + b\right)\right) x \times \text{upstream\_gradient}$$

Consider what happens when the input to a neuron is always positive...

$$f \left( \sum_i w_i x_i + b \right)$$



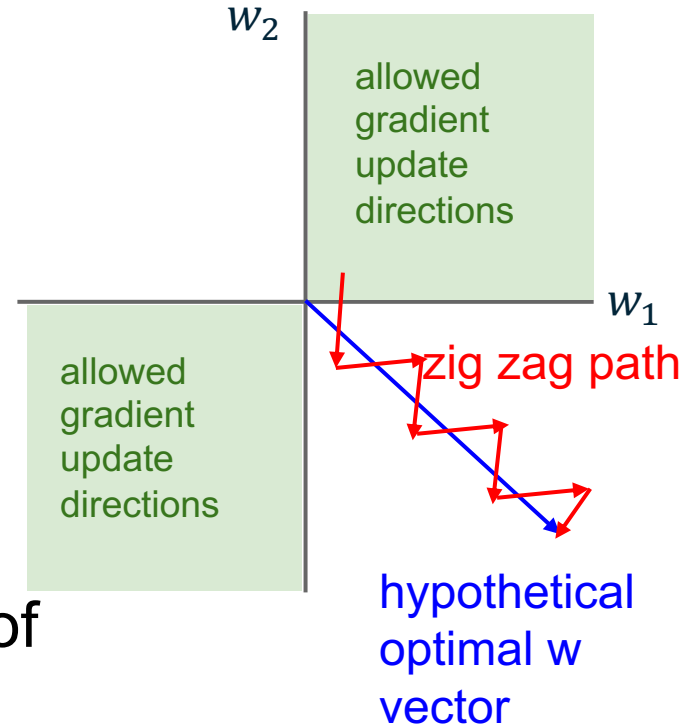
Local gradient cannot change the sign of global gradient.

Consider what happens when the input to a neuron is always positive...

$$f \left( \sum_i w_i x_i + b \right)$$

Local gradient cannot change the sign of global gradient.

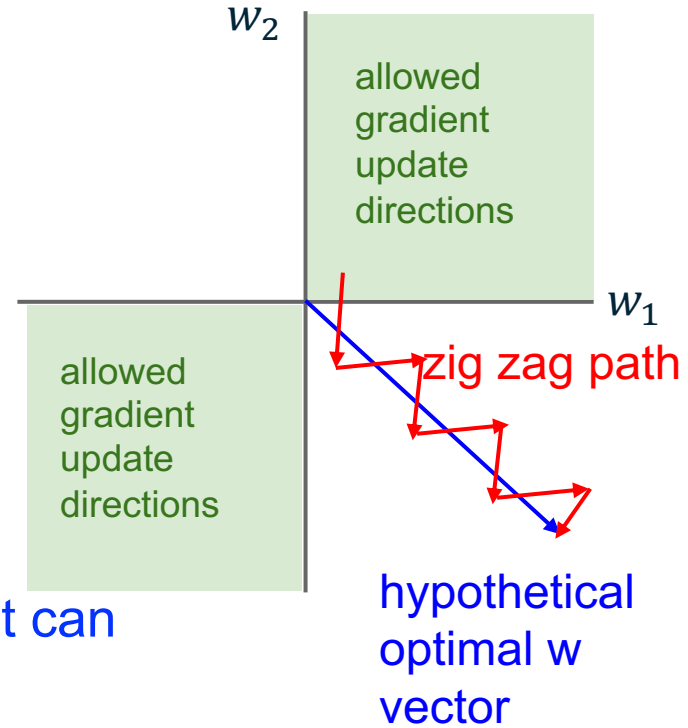
Can easily lead to all-positive or all-negative gradient update (zig-zag).



Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

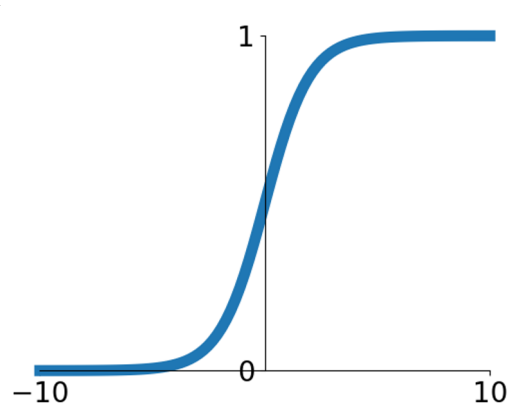
**Remark:** both upstream gradient and local input can change the sign of gradient irrespective of the activation, but having a zero-centered activation function (output spans both positive and negative) can further minimize the “zig-zag” effect



# Activation Functions

$$\sigma(x) = 1/(1 + e^{-x})$$

- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating “firing rate” of a neuron



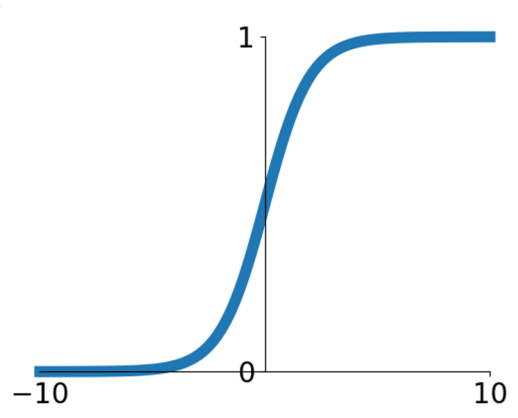
**Sigmoid**

Problems:

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**Sigmoid**

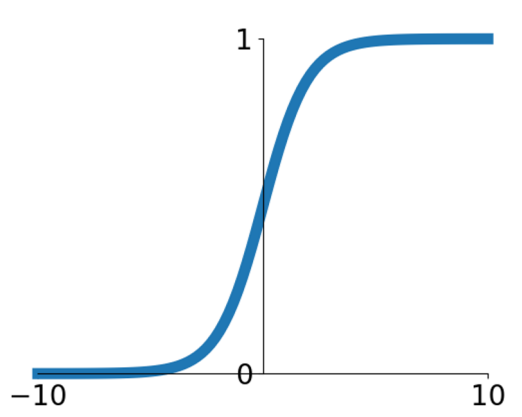
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Problems:

1. Saturated neurons “kill” the gradients
2. Sigmoid outputs are not zero-centered (output does not span both positive and negative)
3.  $\exp()$  is a bit compute expensive



# Activation Functions



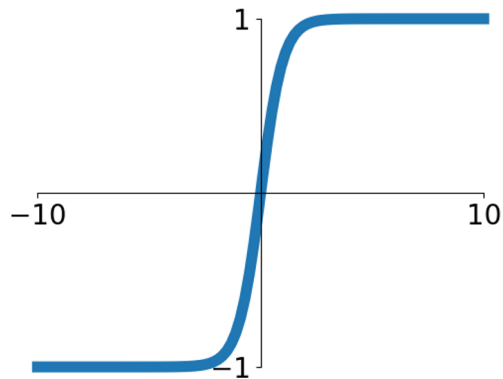
**Sigmoid**

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**Worst problem in practice:  
Saturated neurons “kill” the  
gradients / vanishing gradient**

# Activation Functions

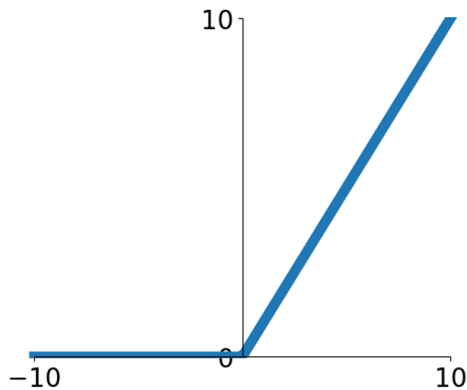


**$\tanh(x)$**

- Squashes numbers to range  $[-1,1]$
- zero centered (nice)
- still kills gradients when saturated :(

[LeCun et al., 1991]

# Activation Functions

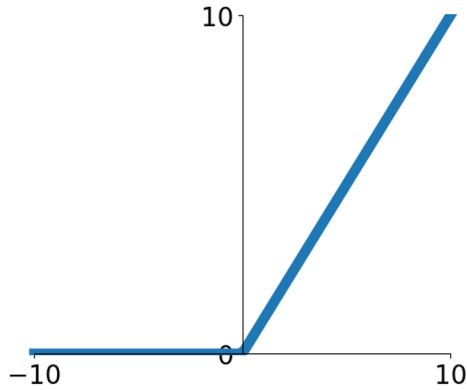


**ReLU**  
(Rectified Linear Unit)

- Computes  $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

[Krizhevsky et al., 2012]

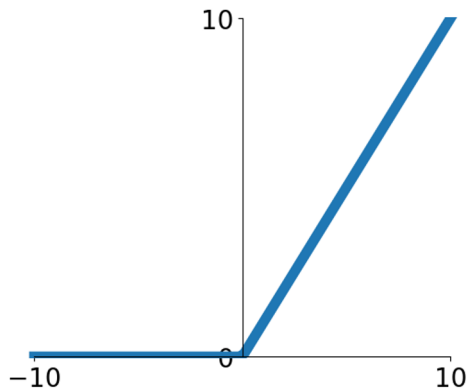
# Activation Functions



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# Activation Functions



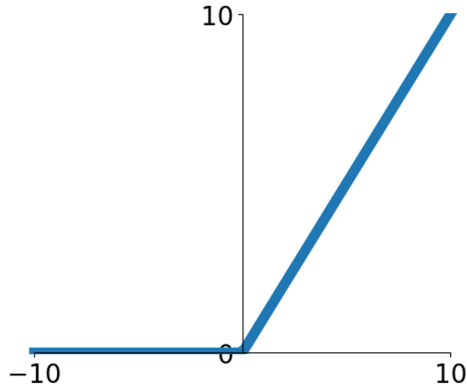
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- An annoyance:

hint: what is the gradient when  $x < 0$ ?

# Activation Functions

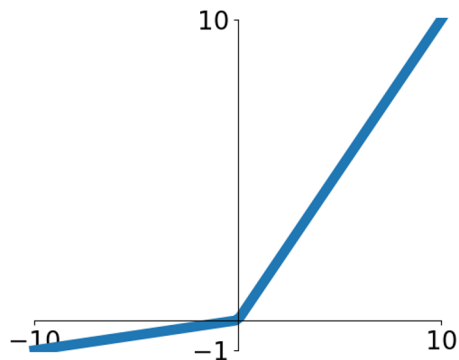


## ReLU (Rectified Linear Unit)

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- Not zero-centered output
- An annoyance:  
hint: what is the gradient when  $x < 0$ ?  
Always 0 -> no update in weights -> stays 0, A.K.A. “dead ReLU”

# Activation Functions

[Mass et al., 2013]  
[He et al., 2015]



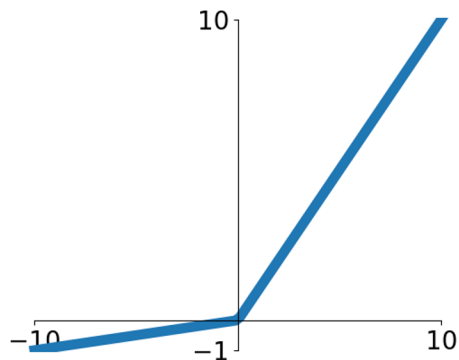
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

## Leaky ReLU

$$f(x) = \max(0.01x, x)$$

# Activation Functions

[Mass et al., 2013]  
[He et al., 2015]



## Leaky ReLU

$$f(x) = \max(0.01x, x)$$

- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

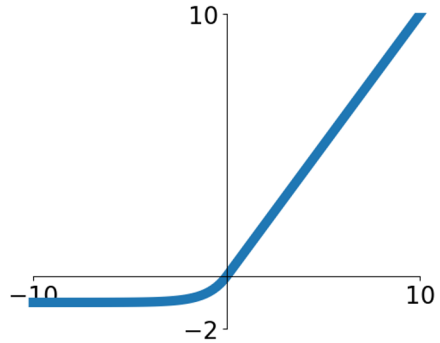
## Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into  $\alpha$   
(parameter)



## Exponential Linear Units (ELU)

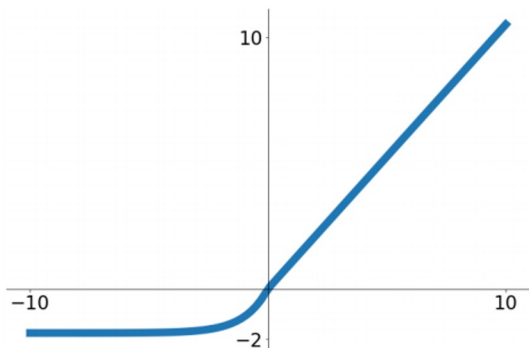


$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha (\exp(x) - 1) & \text{if } x \leq 0 \end{cases}$$

(Alpha default = 1)

- All benefits of ReLU
- Negative saturation encodes presence of features (all goes to  $-\alpha$ ), not magnitude
- Similar in backprop ( $\alpha e^x$  when  $x$  is negative)
- Compared with Leaky ReLU: smooth gradient at 0 (no kink), better optimization landscape

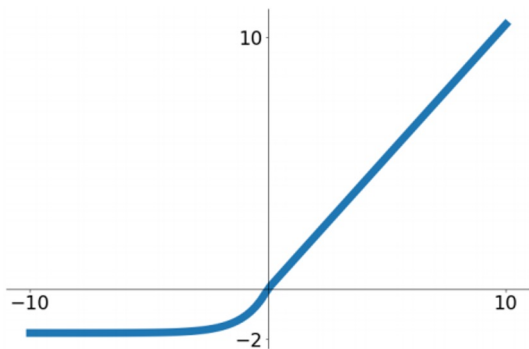
## Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda \alpha (e^x - 1) & \text{otherwise} \end{cases}$$

- Scaled version of ELU that works better for deep networks
- “Self-normalizing” property: under certain conditions, the output of a feedforward network stays around zero-mean and unit variance

## Scaled Exponential Linear Units (SELU)



$$f(x) = \begin{cases} \lambda x & \text{if } x > 0 \\ \lambda \alpha (e^x - 1) & \text{otherwise} \end{cases}$$

$$\alpha = 1.6732632423543772848170429916717$$
$$\lambda = 1.0507009873554804934193349852946$$

- Scaled version of ELU that works better for deep networks
- “Self-normalizing” property: under certain conditions, the output of a feedforward network stays around zero-mean and unit variance

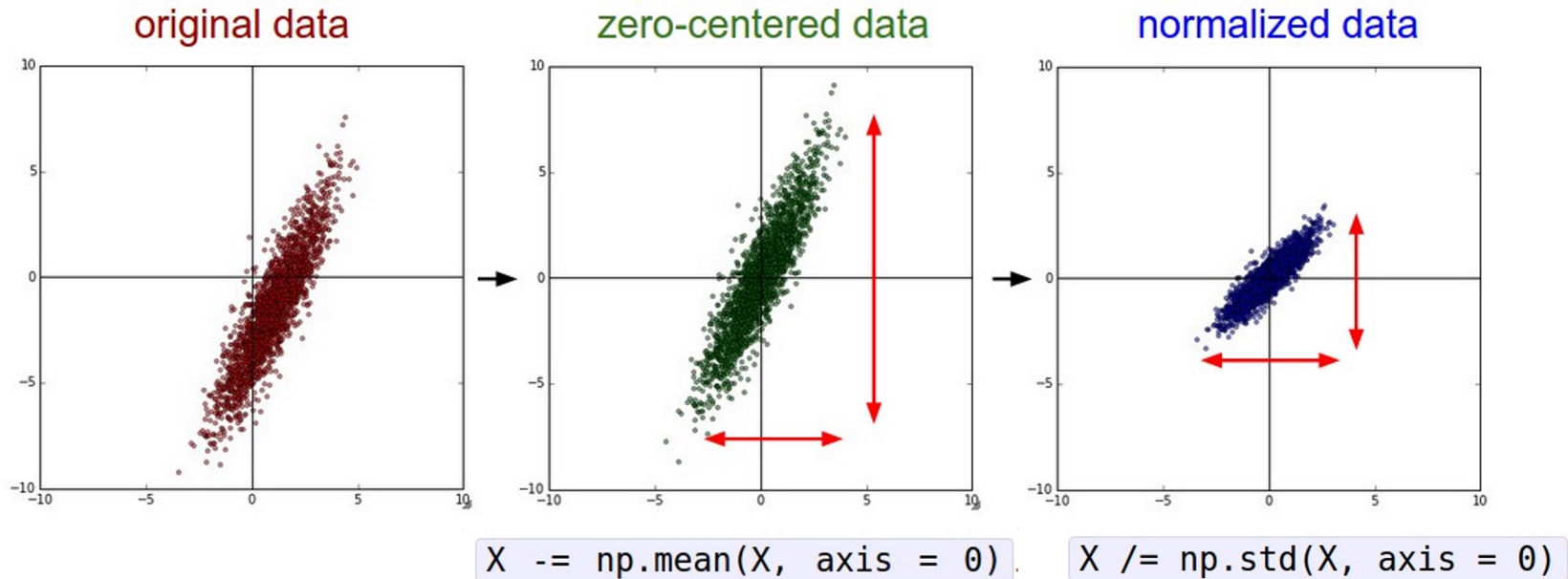
(Klambauer et al, Self-Normalizing Neural Networks, ICLR 2017)

## TLDR: In practice:

- Many possible choices beyond what we've talked here, but ...
- Use **ReLU**. Be careful with your learning rates
- Try out **Leaky ReLU / ELU / SELU**
  - To squeeze out some marginal gains
- Don't use **sigmoid** or **tanh**

# Data Preprocessing

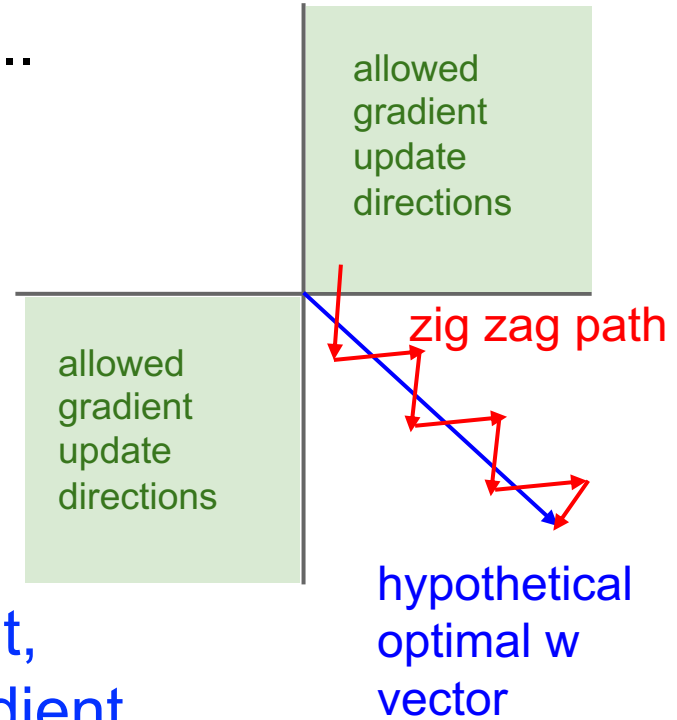
# Data Preprocessing



(Assume  $X$  [NxD] is data matrix,  
each example in a row)

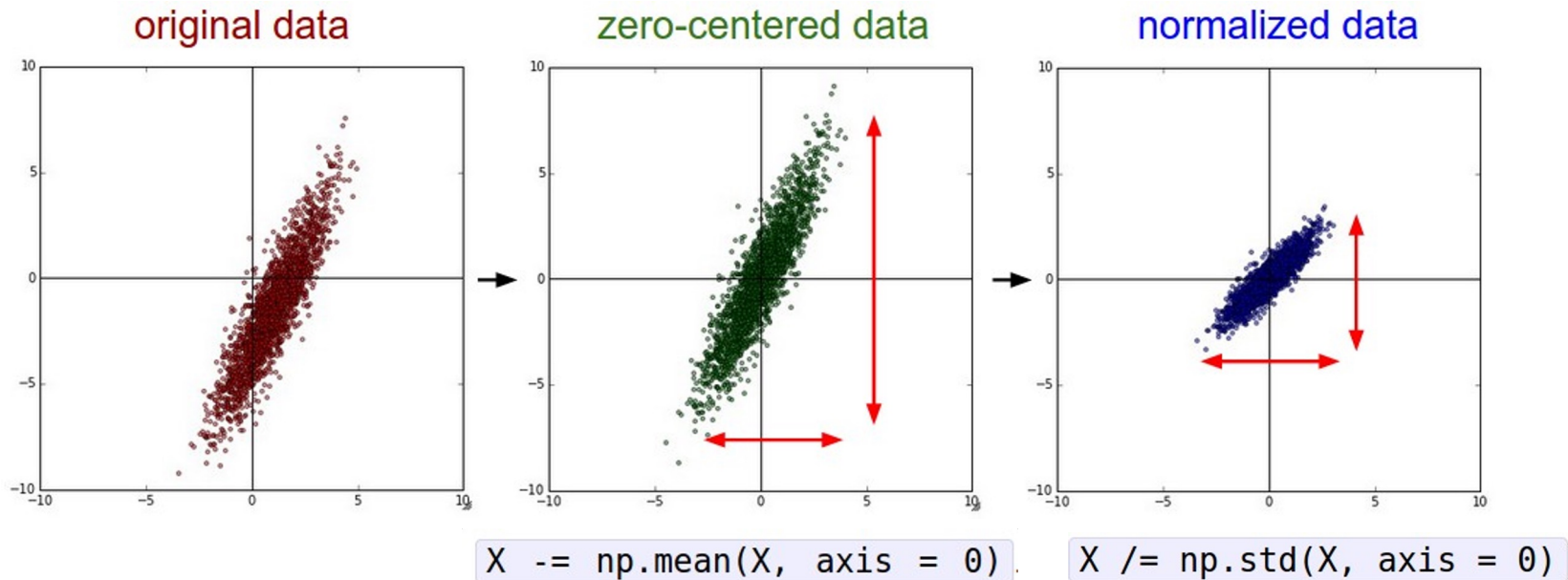
Remember: Consider what happens when the input to a neuron is always positive...

$$f \left( \sum_i w_i x_i + b \right)$$



In addition to upstream and local gradient, input also determines the sign of the gradient. To reduce biases in gradient, we want the input to span both positive and negative value

# Data Preprocessing

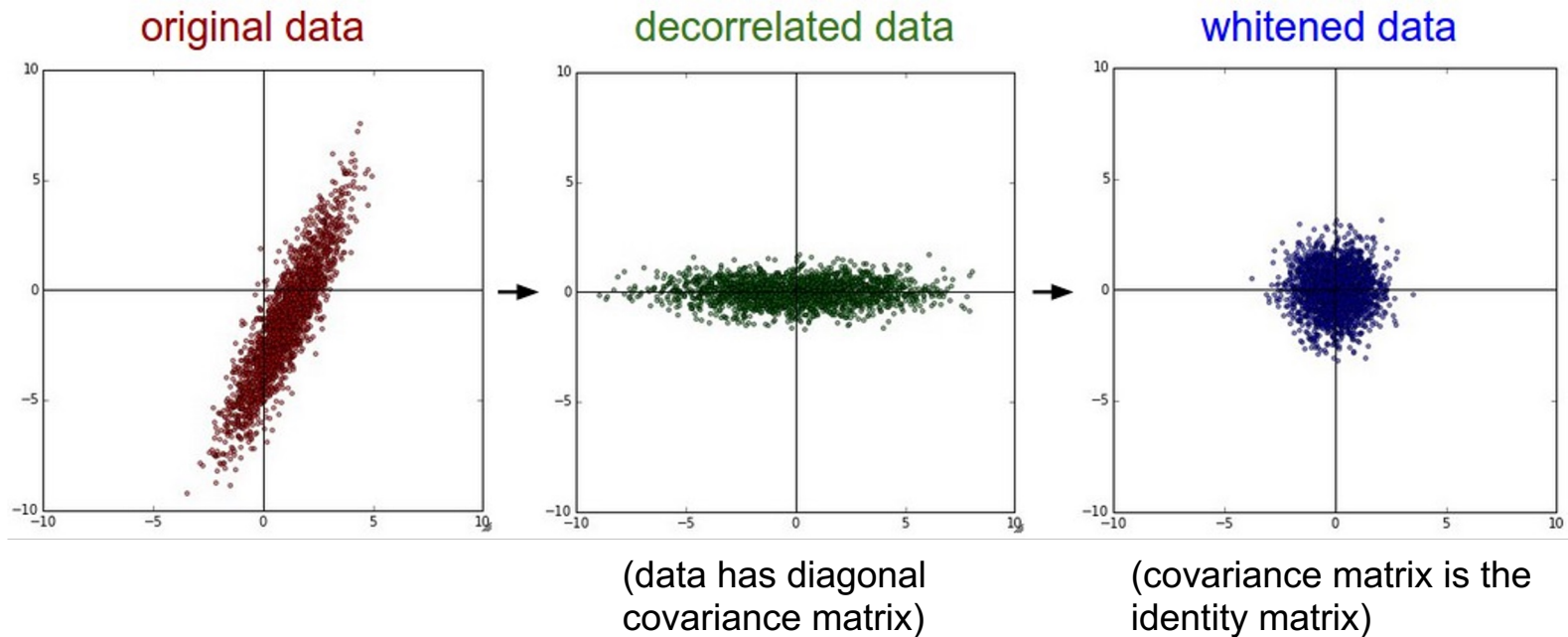


(Assume  $X$  [NxD] is data matrix, each example in a row)



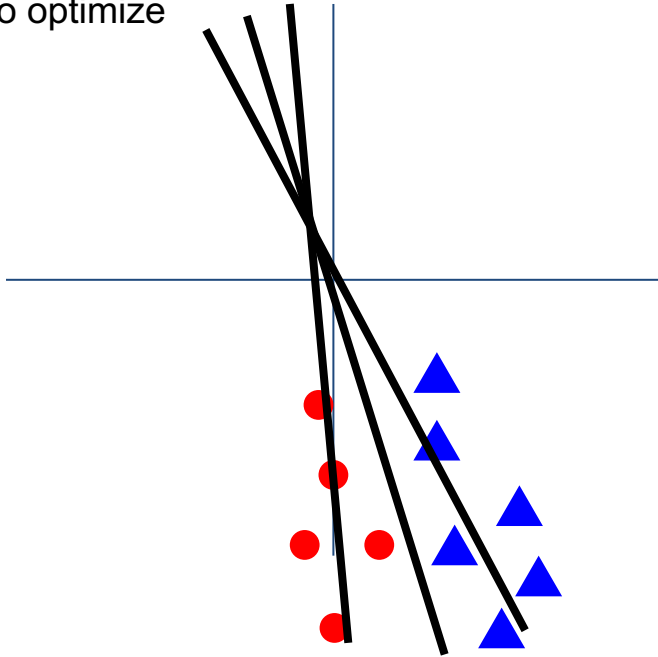
# Data Preprocessing

In practice, you could also **PCA** and **Whitening** of the data

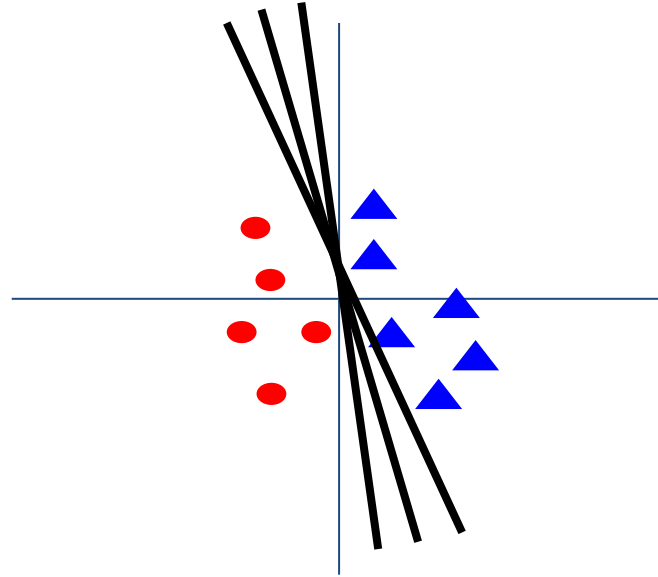


# Data Preprocessing

**Before normalization:** classification loss very sensitive to changes in weight matrix; hard to optimize



**After normalization:** less sensitive to small changes in weights; easier to optimize



# Examples: images

e.g. consider CIFAR-10 example with [32,32,3] images

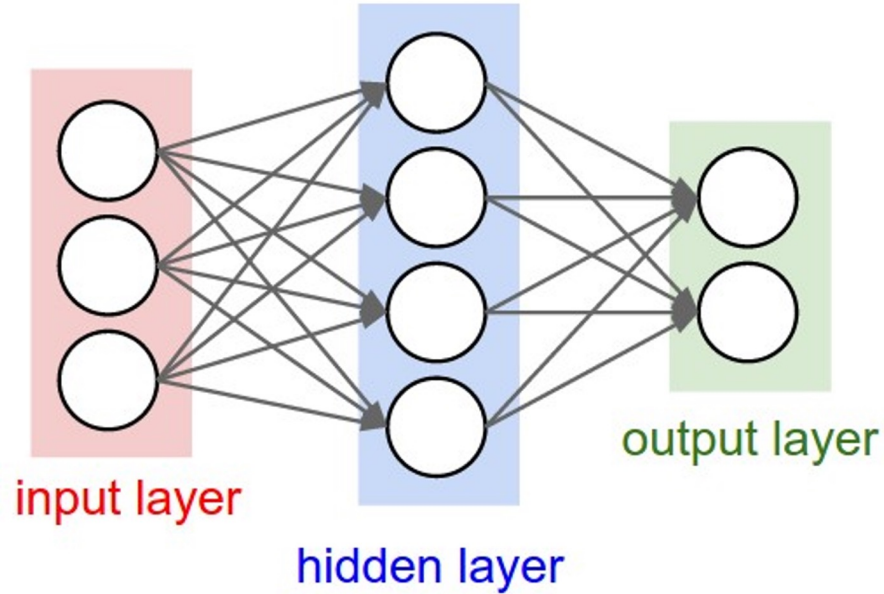
- Subtract the per-pixel mean (e.g. AlexNet)  
(mean image = [32,32,3] array)
- Subtract per-channel mean (e.g. VGGNet)  
(mean along each channel = 3 numbers,)
- Subtract per-channel mean and  
Divide by per-channel std (e.g. ResNet)  
(mean along each channel = 3 numbers)

# Examples: other domains

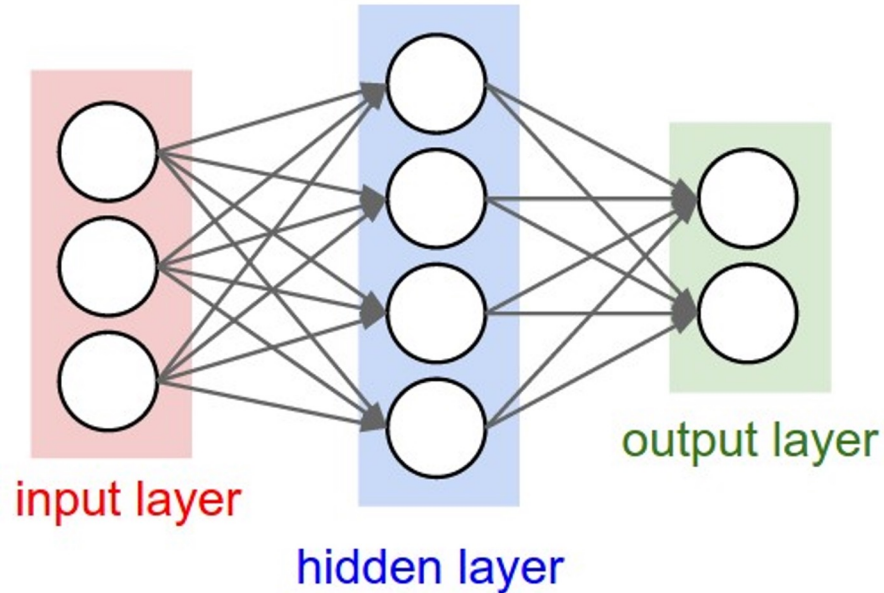
- **Natural language processing:** Normalize word embeddings like Word2Vec or GloVe vectors so that they have a unit norm
- **Graph Neural Networks (GNN):** the feature vector of a node might be scaled by the inverse of its degree or the square root of its degree.
- **Audio data:** Spectral normalize waveforms to ensure that the frequency components are on a similar scale.
- **Reinforcement learning:** reward can be normalized to have zero mean and unit variance to stabilize learning.

# Weight Initialization

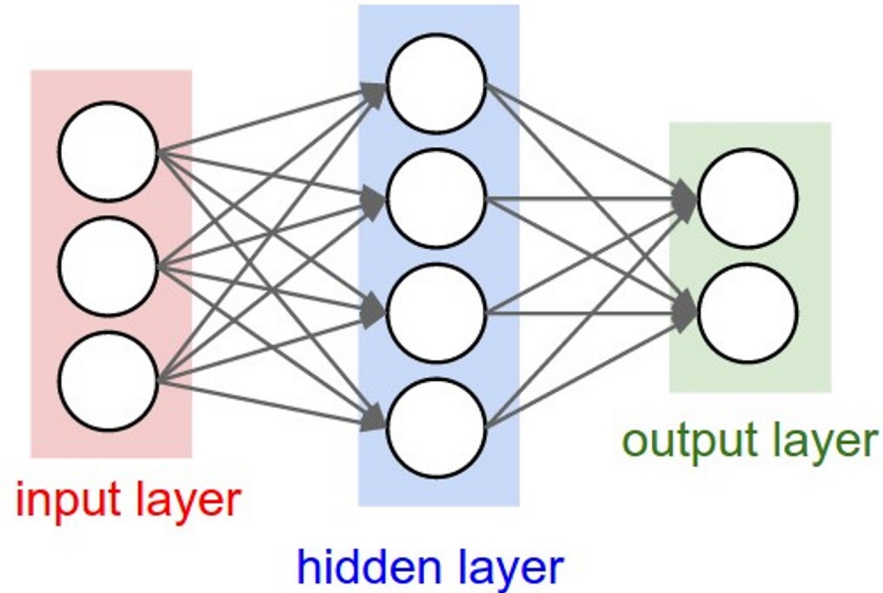
- Q: what happens when  $W$ =same initial value is used?



- Q: what happens when  $W$ =same initial value is used?
- A: All output will be the same!  $w_1^T x = w_2^T x$  if  $w_1 = w_2$



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- A: All output will be the same!  $w_1^T x = w_2^T x$  if  $w_1 = w_2$
- Want to **maintain variance** through the layers.





- First idea: **Small random numbers**  
(gaussian with zero mean and 1e-2 standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

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(gaussian with zero mean and  $1e-2$  standard deviation)

```
W = 0.01 * np.random.randn(Din, Dout)
```

Works ~okay for small networks, but problems with deeper networks.

# Weight Initialization: Activation statistics

```
dims = [4096] * 7      Forward pass for a 6-layer
hs = []               net with hidden size 4096
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.01 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

What will happen to the activations for the last layer?

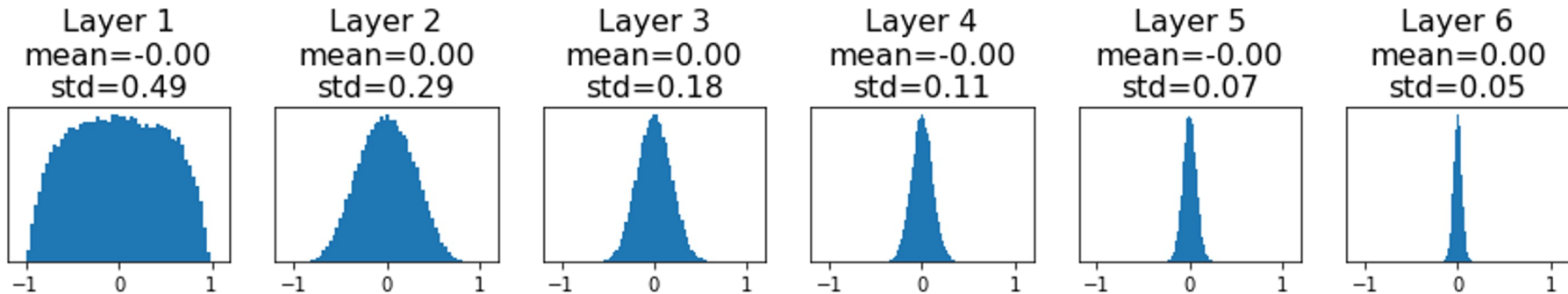
# Weight Initialization: Activation statistics

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    hs.append(x)
```

All activations tend to zero for deeper network layers

**Q:** What do the gradients  $dL/dW$  look like?

**Hint:**  $\frac{\partial L}{\partial w} = x^T \left( \frac{\partial L}{\partial y} \right)$



Visualize distribution of activations

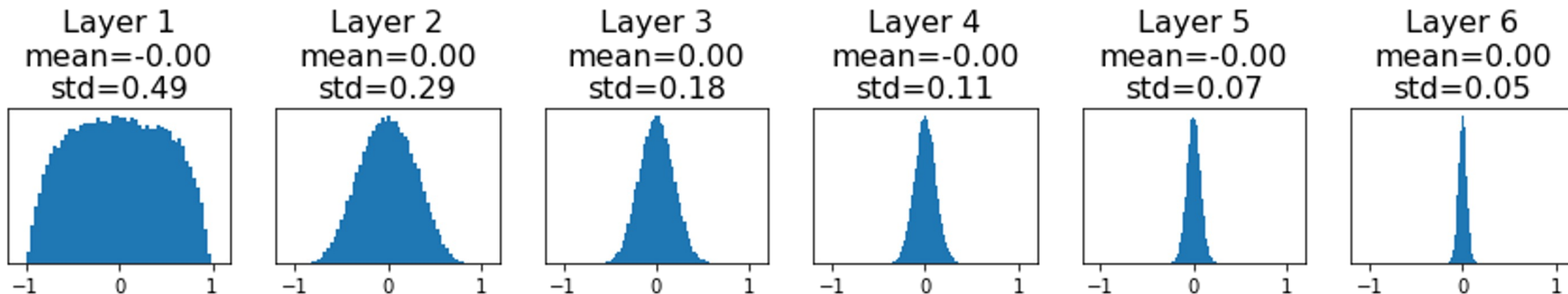
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All activations tend to zero for deeper network layers

**Q:** What do the gradients  $dL/dW$  look like?

**A:** All zero, no learning =(



Visualize distribution of activations

# Weight Initialization: Activation statistics

```
dims = [4096] * 7    Increase std of initial
hs = []             weights from 0.01 to 0.05
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = 0.05 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

Initialize with higher values

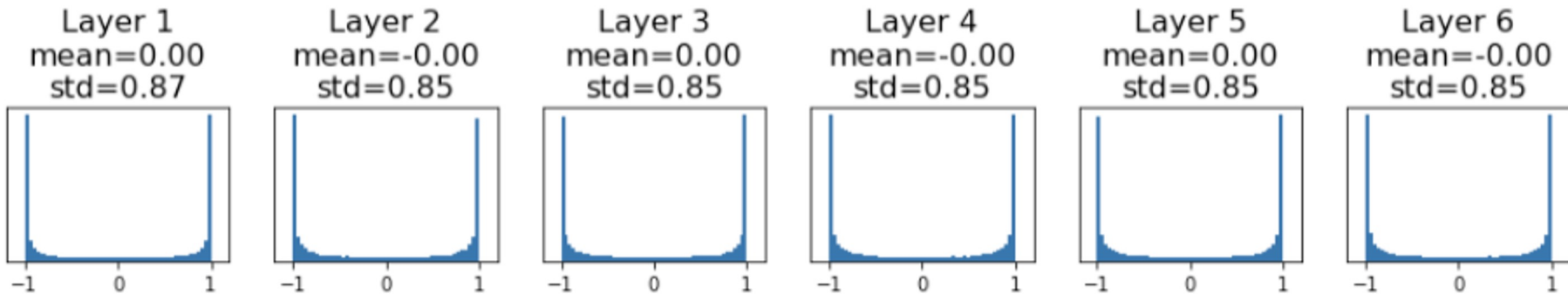
What will happen to the activations for the last layer?

# Weight Initialization: Activation statistics

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    W = 0.05 * np.random.randn(Din, Dout)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

All activations saturate

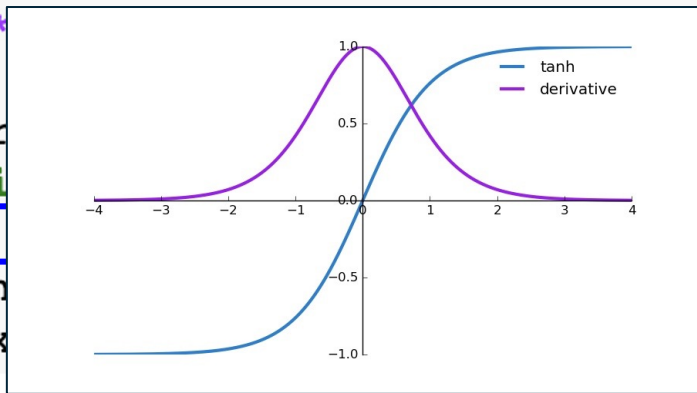
**Q: What do the gradients look like?**



Visualize distribution of activations

# Weight Initialization: Activation statistics

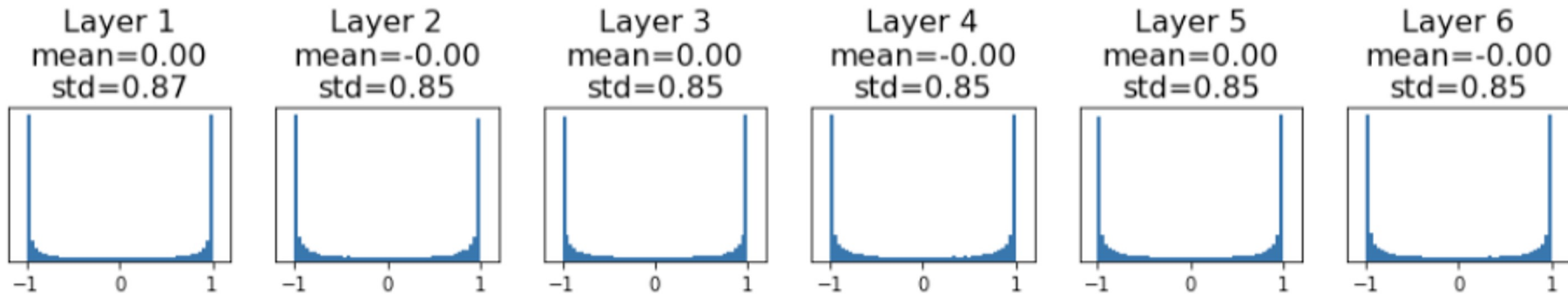
```
dims = [4096] *  
hs = []  
x = np.random.r  
for Din, Dout i  
    W = 0.05 *  
    x = np.tanh  
    hs.append(x
```



All activations saturate

**Q:** What do the gradients look like?

**A:** Local gradients all zero, no learning =(



Visualize distribution of activations



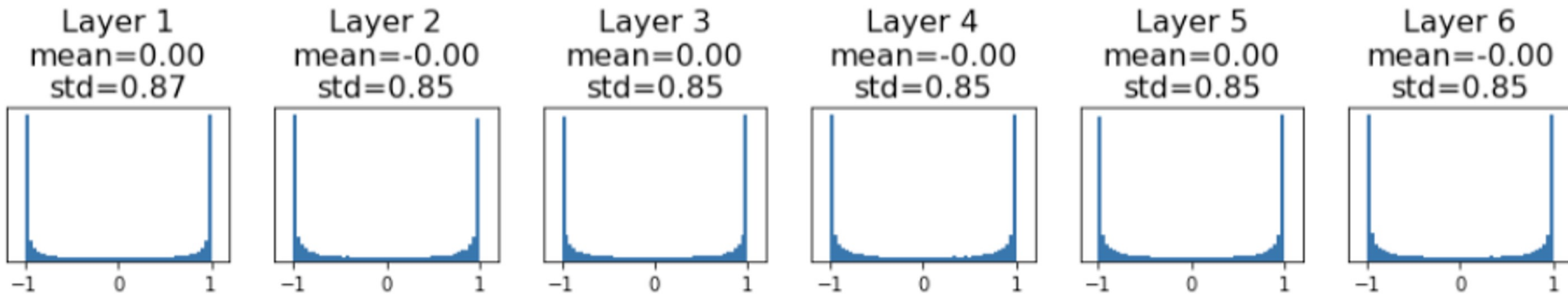
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All activations saturate

**Q:** What do the gradients look like?

*More generally, gradient explosion (high  $w \rightarrow$  high output  $\rightarrow$  high gradient).*



Visualize distribution of activations

# Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7           “Xavier” initialization:
hs = []                     std = 1/sqrt(Din)
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

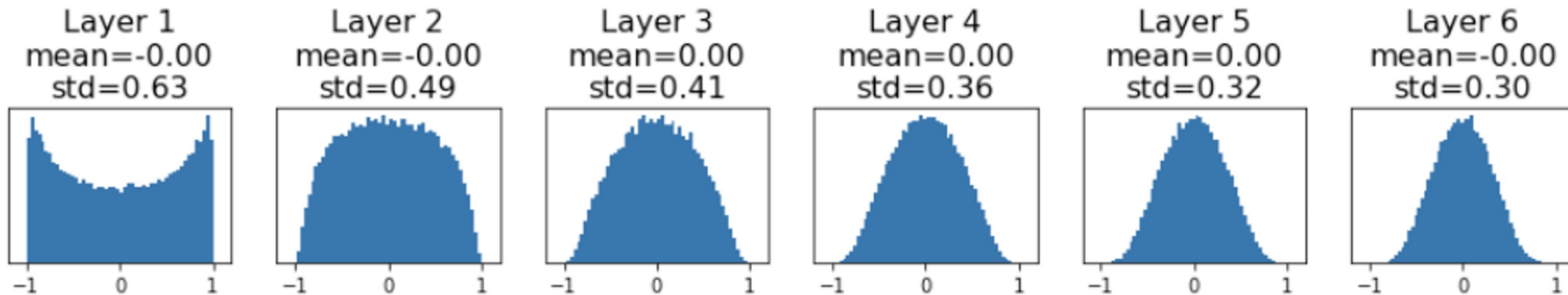
Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

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“Xavier” initialization:  
std =  $1/\sqrt{D_{in}}$

“Just right”: Activations are nicely scaled for all layers!



Glorot and Bengio, “Understanding the difficulty of training deep feedforward neural networks”, AISTAT 2010

Visualize distribution of activations

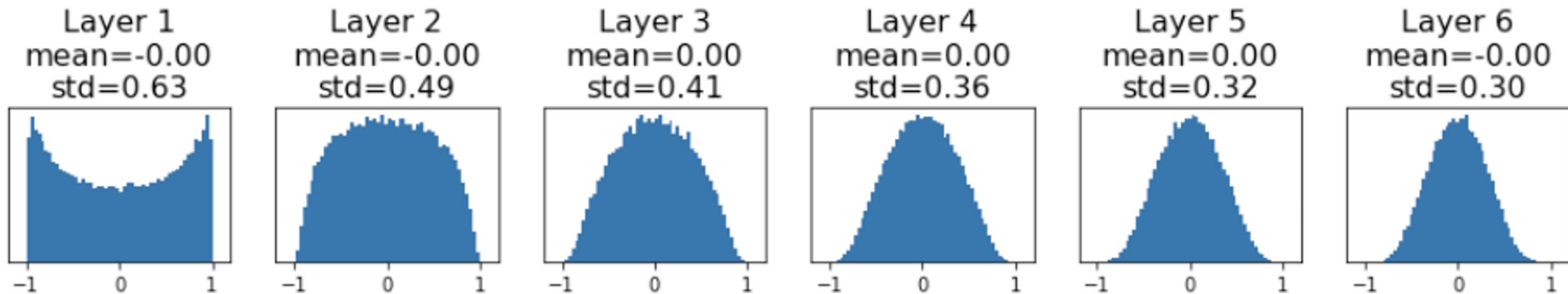
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**Let:**  $y = x_1W_1 + x_2W_2 + \dots + x_{D_{in}}W_{D_{in}}$

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**We want:**  $\text{Var}(y) = \text{Var}(x_i)$

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**We want:**  $\text{Var}(y) = \text{Var}(x_i)$

$\text{Var}(y) = \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{D_{in}}w_{D_{in}})$   
[substituting value of  $y$ ]



# Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
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**Let:**  $y = x_1w_1 + x_2w_2 + \dots + x_{D_{in}}w_{D_{in}}$

**Assume:**  $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{D_{in}})$

**We want:**  $\text{Var}(y) = \text{Var}(x_i)$

$\text{Var}(y) = \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{D_{in}}w_{D_{in}})$

$= \sum \text{Var}(x_iw_i) = D_{in} \text{Var}(x_iw_i)$

[Assume all  $x_i, w_i$  are iid]  $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$

# Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
    hs.append(x)
```

“Xavier” initialization:  
std = 1/sqrt(Din)

“Just right”: Activations are nicely scaled for all layers!

For conv layers, Din is filter\_size<sup>2</sup> \* input\_channels

**Let:**  $y = x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}$

**Assume:**  $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{Din})$

**We want:**  $\text{Var}(y) = \text{Var}(x_i)$

$$\begin{aligned}\text{Var}(y) &= \text{Var}(x_1w_1 + x_2w_2 + \dots + x_{Din}w_{Din}) \\ &= \text{Din} \text{Var}(x_iw_i) \\ &= \text{Din} \text{Var}(x_i) \text{Var}(w_i)\end{aligned}$$

[Assume all  $x_i, w_i$  are zero mean]

$$\begin{aligned}\text{Var}(XY) &= E(X^2Y^2) - (E(XY))^2 = \text{Var}(X)\text{Var}(Y) + \text{Var}(X)(E(Y))^2 \\ &\quad + \text{Var}(Y)(E(X))^2\end{aligned}$$

# Weight Initialization: “Xavier” Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.tanh(x.dot(W))
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```

“Xavier” initialization:  
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For conv layers,  $D_{in}$  is  $\text{filter\_size}^2 * \text{input\_channels}$

**Let:**  $y = x_1 w_1 + x_2 w_2 + \dots + x_{D_{in}} w_{D_{in}}$

**Assume:**  $\text{Var}(x_1) = \text{Var}(x_2) = \dots = \text{Var}(x_{D_{in}})$

**We want:**  $\text{Var}(y) = \text{Var}(x_i)$

$$\begin{aligned}\text{Var}(y) &= \text{Var}(x_1 w_1 + x_2 w_2 + \dots + x_{D_{in}} w_{D_{in}}) \\ &= D_{in} \text{Var}(x_i w_i) \\ &= D_{in} \text{Var}(x_i) \text{Var}(w_i) \\ &[\text{Assume all } x_i, w_i \text{ are iid}]\end{aligned}$$

So,  $\text{Var}(y) = \text{Var}(x_i)$  only when  $\text{Var}(w_i) = 1/D_{in}$

# Weight Initialization: What about ReLU?

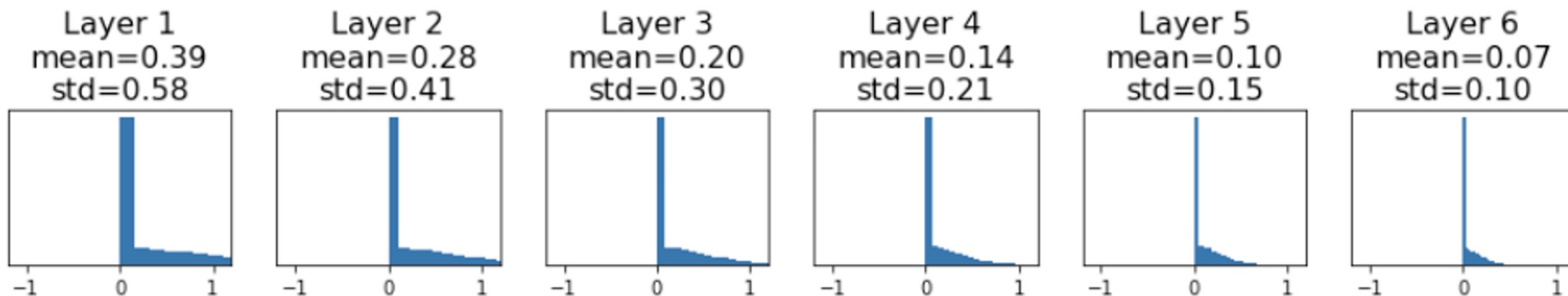
```
dims = [4096] * 7      Change from tanh to ReLU
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

# Weight Initialization: What about ReLU?

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dims = [4096] * 7      Change from tanh to ReLU
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x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) / np.sqrt(Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

Xavier assumes zero centered activation function

Activations collapse to zero again, no learning =(



Visualize distribution of activations

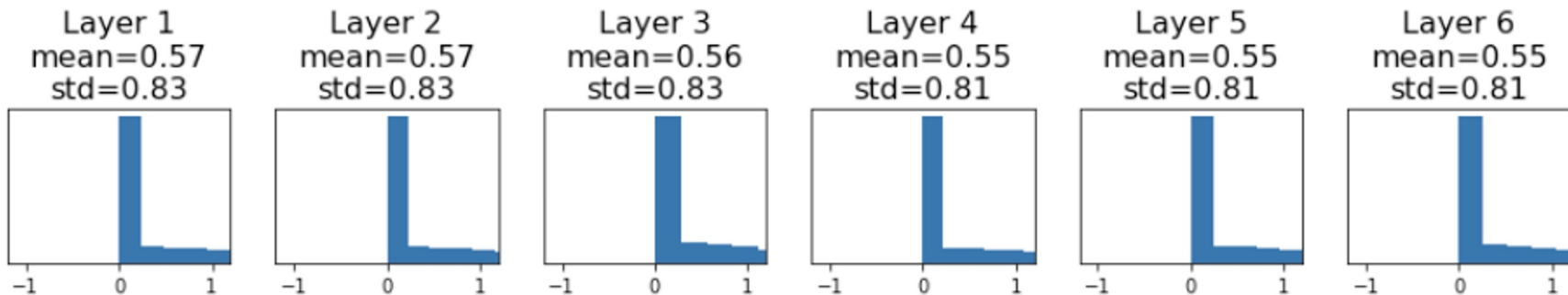
# Weight Initialization: Kaiming / MSRA Initialization

```
dims = [4096] * 7
hs = []
x = np.random.randn(16, dims[0])
for Din, Dout in zip(dims[:-1], dims[1:]):
    W = np.random.randn(Din, Dout) * np.sqrt(2/Din)
    x = np.maximum(0, x.dot(W))
    hs.append(x)
```

ReLU correction:  $\text{std} = \sqrt{2 / \text{Din}}$

Issue: Half of the activation get killed.

Solution: make the non-zero output variance twice as large as input



He et al, "Delving Deep into Rectifiers: Surpassing Human-Level Performance on ImageNet Classification", ICCV 2015

Visualize distribution of activations

# Proper initialization is an active area of research...

***Understanding the difficulty of training deep feedforward neural networks***

by Glorot and Bengio, 2010

***Exact solutions to the nonlinear dynamics of learning in deep linear neural networks*** by Saxe et al, 2013

***Random walk initialization for training very deep feedforward networks*** by Sussillo and Abbott, 2014

***Delving deep into rectifiers: Surpassing human-level performance on ImageNet classification*** by He et al., 2015

***Data-dependent Initializations of Convolutional Neural Networks*** by Krähenbühl et al., 2015

***All you need is a good init***, Mishkin and Matas, 2015

***Fixup Initialization: Residual Learning Without Normalization***, Zhang et al, 2019

***The Lottery Ticket Hypothesis: Finding Sparse, Trainable Neural Networks***, Frankle and Carbin, 2019

# Summary

## **Training** Deep Neural Networks

- Details of the non-linear activation functions
  - Sigmoid, Tanh, ReLU, LeakyRELU, ELU, SELU
- Data normalization
  - Zero-centering, decorrelation, image normalization
- Weight Initialization
  - Constant init, random init, Xavier Init, Kaiming Init



# Next time:

## **Training Deep Neural Networks**

- Details of the non-linear activation functions
- Data normalization
- Weight Initialization
- **Batch Normalization**
- **Advanced Optimization**
- **Regularization**
- Data Augmentation
- Transfer learning
- Hyperparameter Tuning
- Model Ensemble