CS 4803-DL / 7643-A: LECTURE 23
DANFEI XU

Topics:

- Reinforcement Learning Part 1
  - **Markov Decision Processes**
  - **Value Iteration**
  - **(Deep) Q Learning**

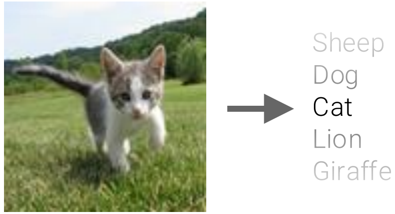
Administrative

- Project Report Due 11/30
- Project Presentation Due 12/1

Reinforcement Learning Introduction

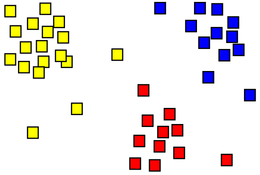
Supervised Learning

- Train Input: $\{X, Y\}$
- Learning output: $f : X \rightarrow Y$, $P(y|x)$
- e.g. classification



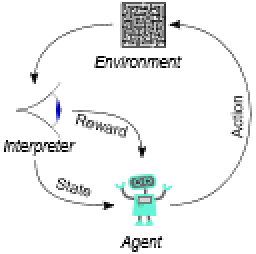
Unsupervised Learning

- Input: $\{X\}$
- Learning output: $P(x)$
- Example: Clustering, density estimation, generative modeling



Reinforcement Learning

- Evaluative feedback in the form of **reward**
- No supervision on the right action



Types of Machine Learning

Decision Making

- **Interactive Environment:** Unlike other ML paradigms, decision making is to act optimally in a dynamic, interactive environment.
- **Feedback Loop:** The agent's actions directly influence the future distribution of inputs, creating a continuous feedback loop.
- **Optimality:** The goal is to learn actions that maximize sum of future rewards, focusing on long-term outcomes.
- **Learn to predict:** The model must be able to predict, either implicitly or explicitly, how the environment changes in response to the agent's actions.

RL: Sequential decision making in an environment with evaluative feedback.

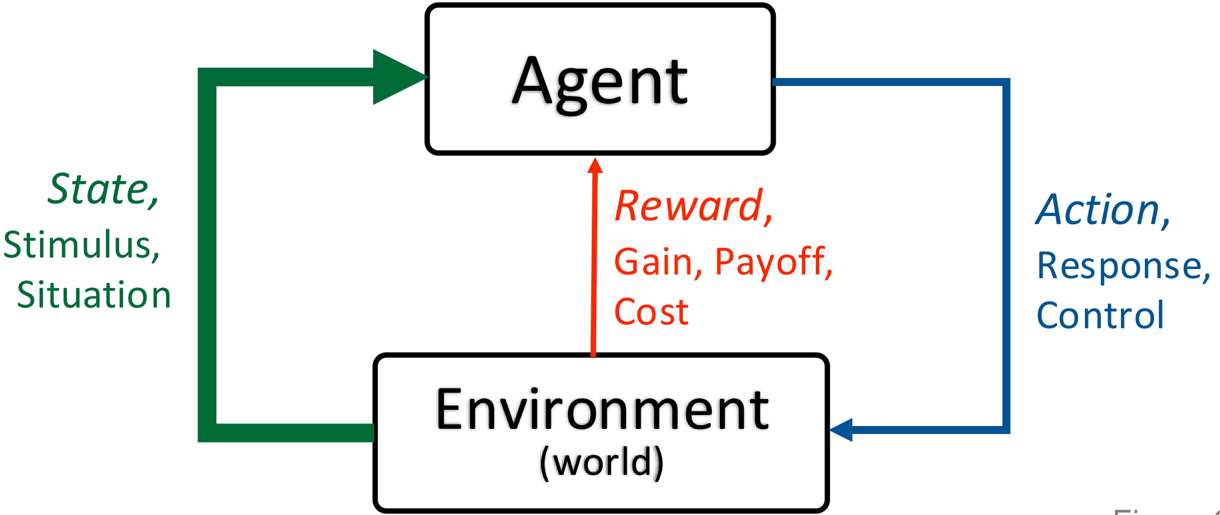
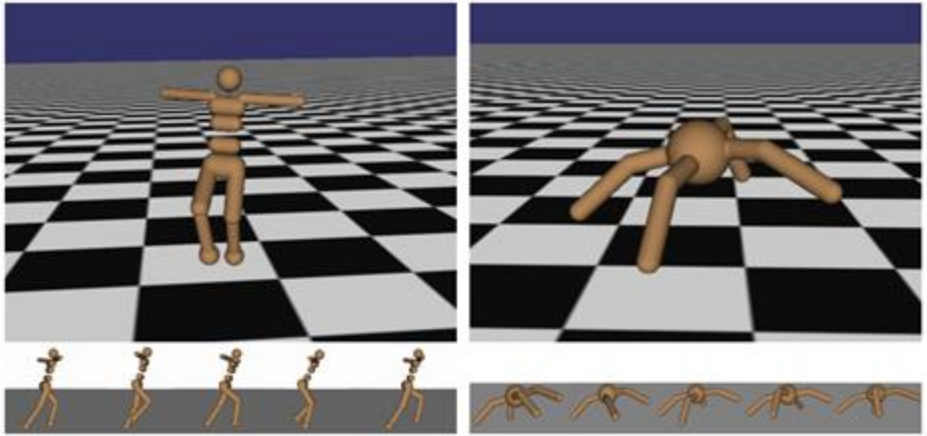


Figure Credit: Rich Sutton

- **Environment** may be unknown, non-linear, stochastic and complex.
- **Agent** learns a **policy** to map states of the environments to actions.
  - Seeking to maximize cumulative reward in the long run.

What is Reinforcement Learning?

Example: Robot Locomotion



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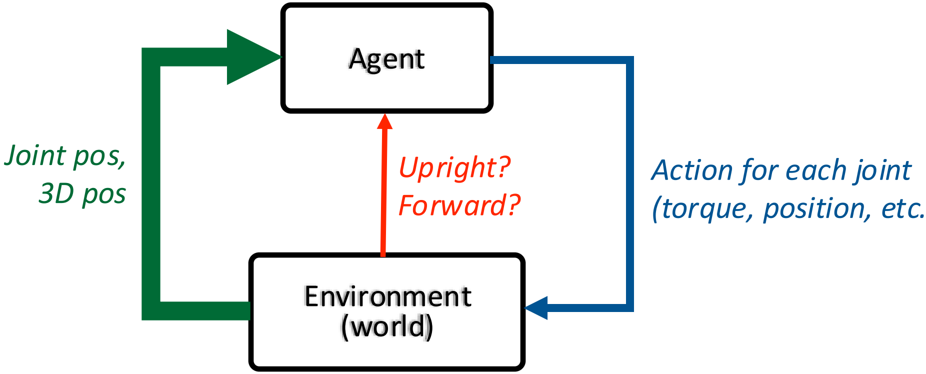
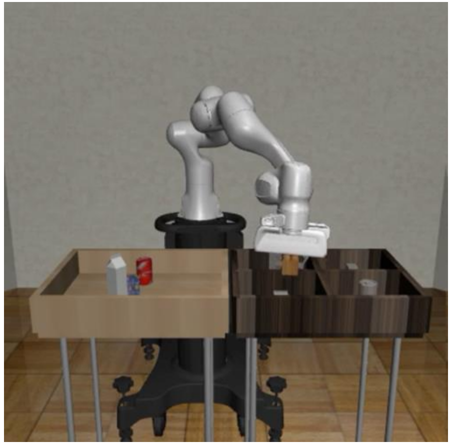
- **Objective**: Make the robot move forward without falling
- **State**: Angle and position of the joints
- **Action**: Torques applied on joints
- **Reward**: +1 at each time step upright and moving forward

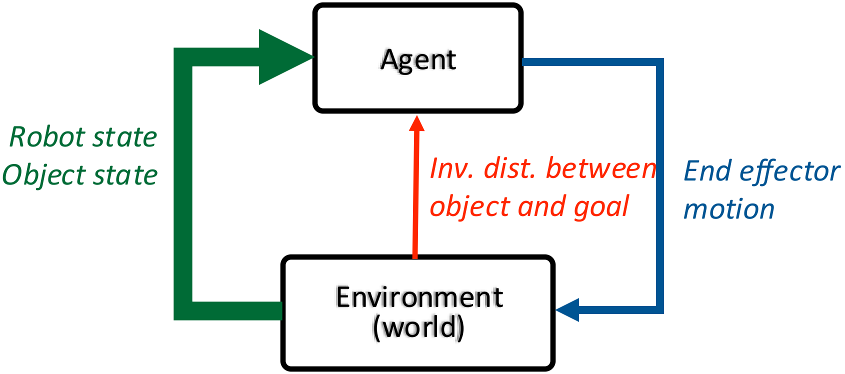


Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Example: Robot Manipulation



- **Objective**: Pick up object and place to sorting bin
- **State**: Pose of the object and the bin, joint state and velocity of robots
- **Action**: End effector motion
- **Reward**: inverse distance between the object and the bin



Example: Atari Games

- **Objective**: Complete the game with the highest score
- **State**: Raw pixel inputs of the game state
- **Action**: Game controls e.g. Left, Right, Up, Down
- **Reward**: Score increase/decrease at each time step

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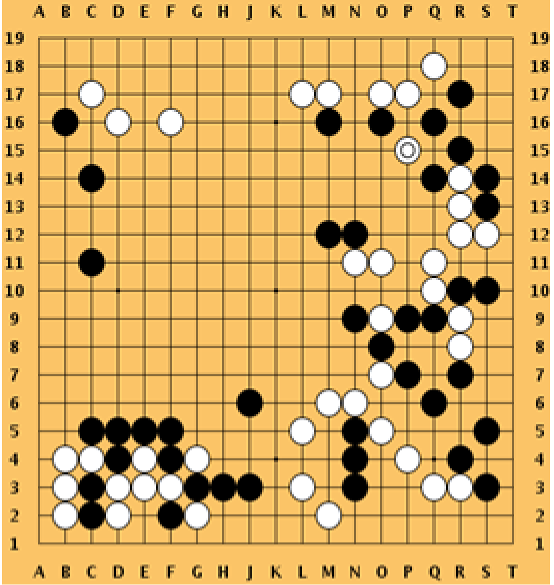
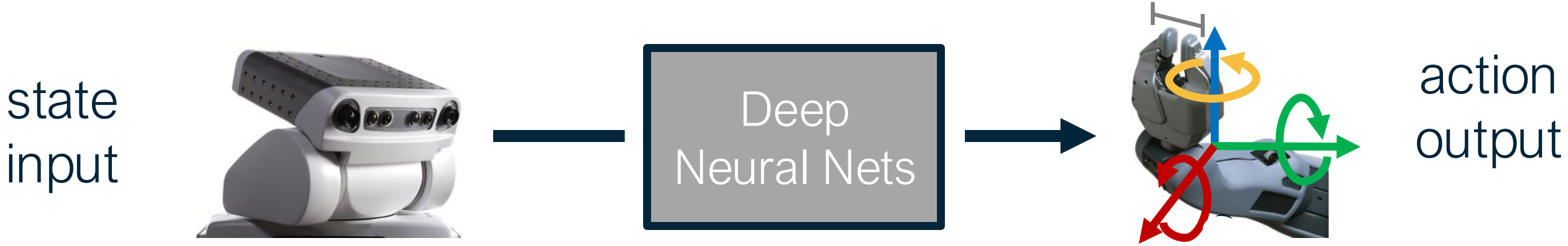
Example: Go



- **Objective**: Defeat opponent
- **State**: Board pieces
- **Action**: Where to put next piece down
- **Reward**: +1 if win at the end of game, 0 otherwise

Slide Credit: Fei-Fei Li, Justin Johnson, Serena Yeung, CS 231n

Deep Learning for Decision Making



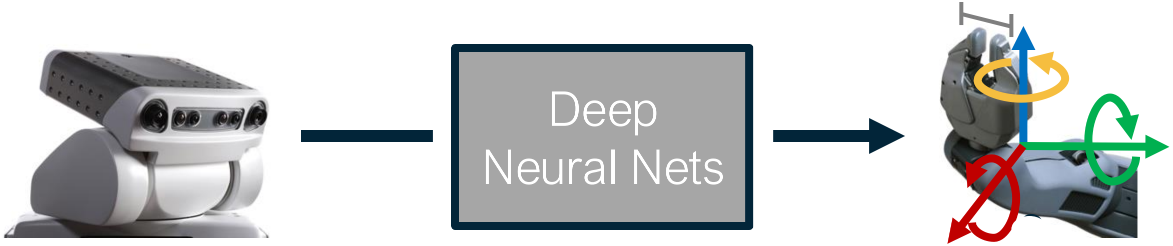
state input

Deep Neural Nets

action output

Deep Learning for Decision Making

state
input

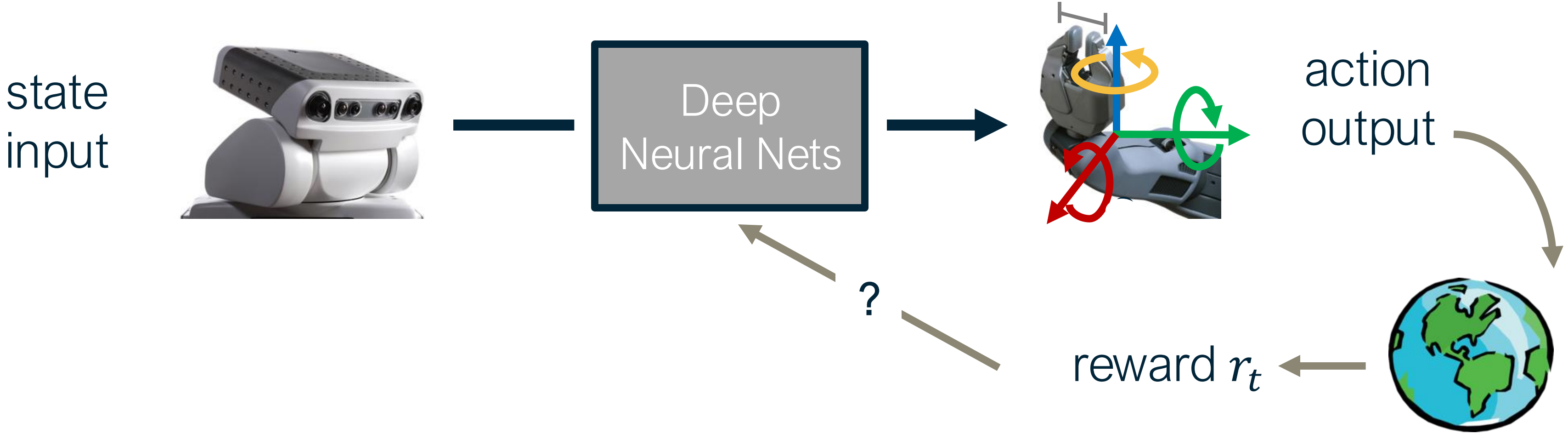


Deep
Neural Nets

action
output

Problem: we don’t know the correct action label to supervise the output!

Deep Learning for Decision Making



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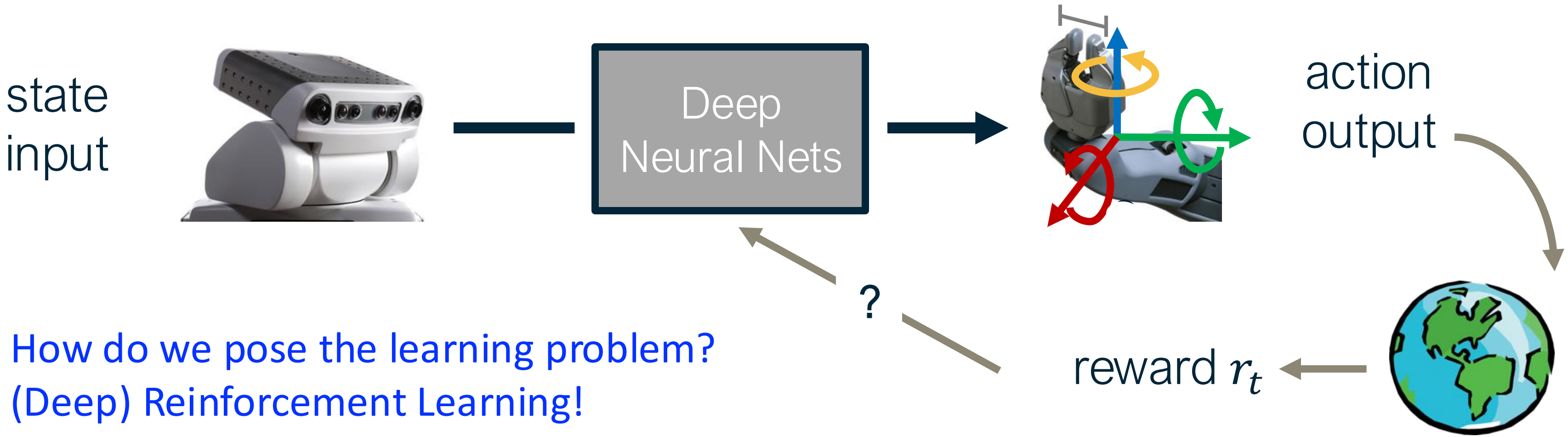
All we know is the step-wise task reward

Deep Learning for Decision Making



How do we pose the learning problem?
(Deep) Reinforcement Learning!

Problem: we don’t know the correct action label to supervise the output!

All we know is the step-wise task reward

Markov Decision Processes

- **MDPs**: Theoretical framework underlying RL

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- An MDP is defined as a tuple $(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

  $\mathcal{S}$: Set of possible states

  $\mathcal{A}$: Set of possible actions

  $\mathcal{R}(s, a, s')$: Distribution of reward

  $\mathbb{T}(s, a, s')$: Transition probability distribution, also written as $p(s'|s, a)$

  γ : Discount factor

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- **Experience**: $\ldots s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}, r_{t+2}, s_{t+2}, \ldots$

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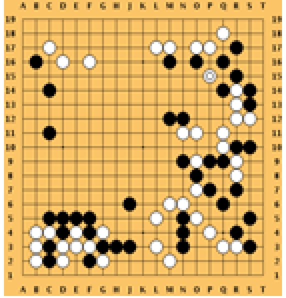
  γ : Discount factor
- **Experience**: $\ldots s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}, r_{t+2}, s_{t+2}, \ldots$
- **Markov property**: Current state completely characterizes state of the environment
- **Assumption**: Most recent observation is a sufficient statistic of history

$$p\left(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, \ldots S_0 = s_0\right) = p\left(S_{t+1} = s' | S_t = s_t, A_t = a_t\right)$$

Fully observed MDP

- Agent receives the true state s_t at time t
- Example: Chess, Go



Partially observed MDP

- Agent perceives its own partial observation o_t of the state s_t at time t, using past states e.g. with an RNN
- Example: Poker, First-person games (e.g. Doom)

Source: https://github.com/mwydmuch/ViZDoom

MDP Variations

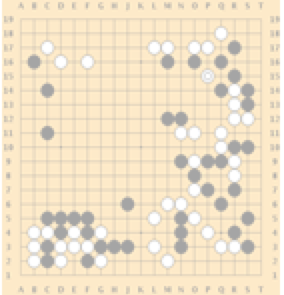
Fully observed MDP

- Agent receives the true state s_t at time t
- Example: Chess, Go

Partially observed MDP

- Agent perceives its own partial observation o_t of the state s_t at time t, using past

We will assume **fully observed MDPs** for this lecture



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MDP Variations

- In **Reinforcement Learning**, we assume an underlying **MDP** with unknown:
  - Transition probability distribution $\mathbb{T}$
  - Reward distribution $\mathcal{R}$

MDP
$(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

MDPs in the context of RL

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  - Transition probability distribution $\mathbb{T}$
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MDP
$(\mathcal{S}, \mathcal{A}, \mathcal{R}, \mathbb{T}, \gamma)$

Put simply: without learning, the agent **doesn't know** how their actions will change the environment and what reward they will receive.

Reinforcement Learning is to learn to act optimally given experience data (transition, reward) from interacting with the environments.

The outcome is a control policy $\pi(a|s)$ that maps a state s to a (good) action a

MDPs in the context of RL

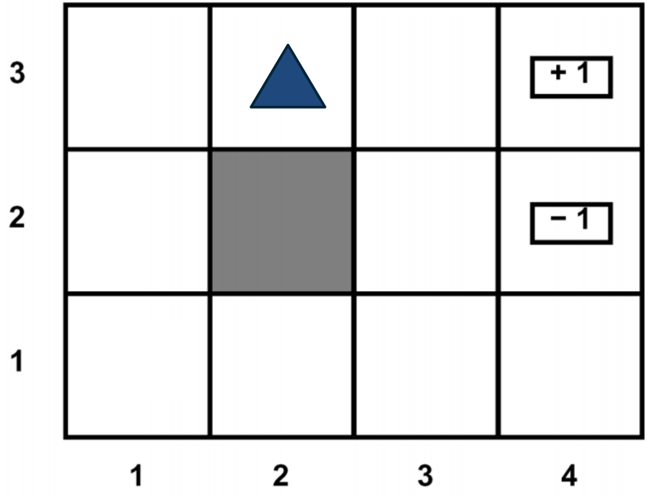
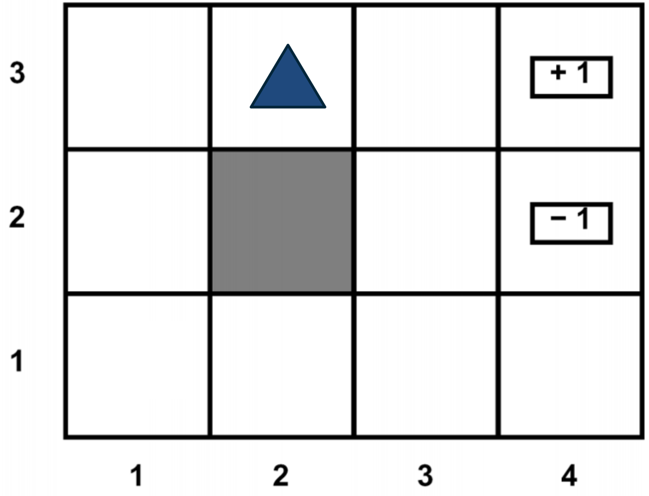


Figure credits: Pieter Abbeel

A Grid World MDP

- Agent lives in a 2D grid environment



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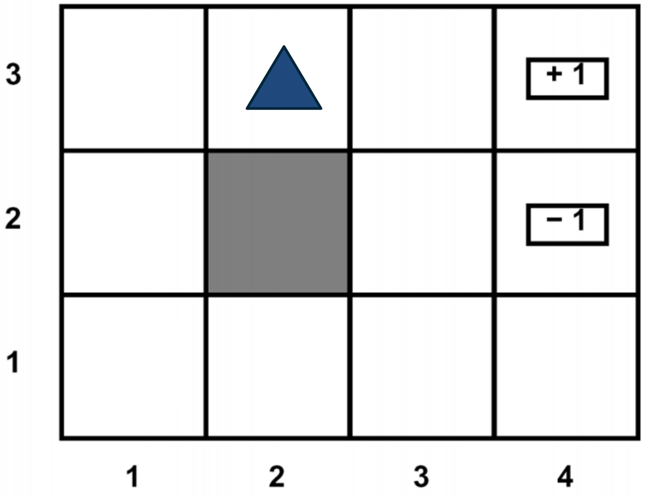
A Grid World MDP

- Agent lives in a 2D grid environment

- State: Agent’s 2D coordinates
- Actions: N, E, S, W
- Rewards: +1/-1 at absorbing states



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- Agent lives in a 2D grid environment

- State: Agent’s 2D coordinates
- Actions: N, E, S, W
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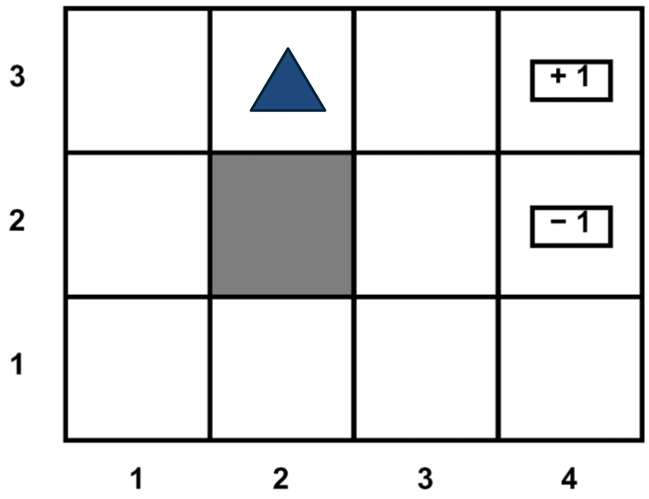
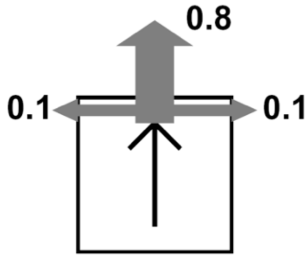
- Walls block agent’s path
- Actions to not always go as planned $T(s, a, s')$
  - 20% chance that agent drifts one cell left or right of direction of motion (except when blocked by wall).





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- Solving MDPs by finding the **best/optimal policy**

Solving MDPs: Optimal policy

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- Solving MDPs by finding the **best/optimal policy**

- Formally, a **policy** is a mapping from states to actions

e.g.

State	Action
A →	2
B →	1

- Solving MDPs by finding the **best/optimal policy**

- Formally, a **policy** is a mapping from states to actions
  - Deterministic $\pi(s) = a$

Solving MDPs: Optimal policy

- Solving MDPs by finding the **best/optimal policy**

- Formally, a **policy** is a mapping from states to actions
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  - Stochastic $\pi(a|s) = \mathbb{P}(A_t = a|S_t = s)$

Solving MDPs: Optimal policy

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  - Maximize **current reward**? Sum of all **future rewards**?

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  - **Discounted sum of future rewards**!
  - Future is inherently uncertain!

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- Formally, the **optimal policy** is defined as:

$$\pi^* = \arg\max_{\pi} \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | \pi\right]$$

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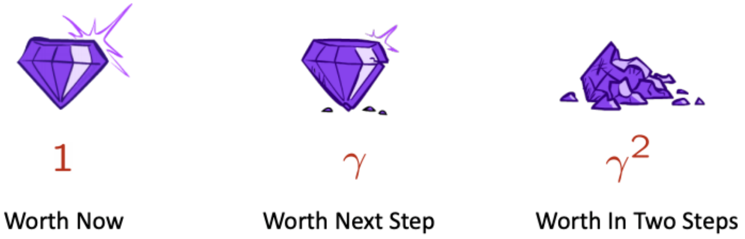
Small γ → near-sighted

Large γ → far-sighted

Future is inherently uncertain!

How much to value future rewards

- Discount factor: γ
- Typically 0.9 - 0.99



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$$\boxed{\boldsymbol{s}_0 \sim p\left(\boldsymbol{s}_0\right), a_t \sim \pi\left(\cdot|\boldsymbol{s}_t\right), \boldsymbol{s}_{t+1} \sim p\left(\cdot|\boldsymbol{s}_t, a_t\right)}$$

Expectation over initial state, actions from policy,
next states from transition distribution

We need a function to quantify the optimality of a policy!

- A **value function** predicts the sum of discounted future reward **for a given policy**

Value Function

- A **value function** predicts the sum of discounted future reward **for a given policy**

- **State** value function / **V**-function / $V : \mathcal{S} \rightarrow \mathbb{R}$
  - How good is this state **under a policy?**
  - Am I likely to win/lose the game from this state (reward-to-go)?

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- **State-Action** value function / **Q**-function / $Q : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}$
  - How good is this state-action pair w.r.t a policy?
  - In this state, what is the impact of this action on my future?

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Value functions are measuring both the quality of a state (state-action pair) and the quality of a policy!

Value Function

- For a policy that produces a trajectory sample $(s_0, a_0, s_1, a_1, s_2 \cdots)$

Value Function

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- The **V-function** of the policy at state s, is the expected cumulative reward from state s:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

Value Function

Georgia Tech

Value Function

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- For a policy that produces a trajectory sample $(s_0, a_0, s_1, a_1, s_2 \cdots)$

- The **Q-function** of the policy at state **s** and action **a**, is the expected cumulative reward upon taking action **a** in state **s** (and following policy thereafter):

$$Q^{\pi}(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

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How do we learn a good policy?

Action-Value Function

- The V and Q functions corresponding to **the optimal policy** $\pi^\star$ **of a given MDP**

$$V^*(s) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, \pi^*\right]$$

$$Q^*(s,a) = \mathbb{E}\left[\sum_{t\geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi^*\right]$$

$$\boxed{V^*(s) = \max_a Q^*(s,a)}$$

Optimal policy from Q value function: $\boxed{\pi^*(s) = \arg\max_a Q^*(s,a)}$

Optimal V & Q functions

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$$\boxed{V^*(s) = \max_a Q^*(s,a)}$$

If we know Q, we have a policy! How do we learn the value functions?

Optimal policy from Q value function: $\boxed{\pi^*(s) = \arg\max_a Q^*(s,a)}$

Optimal V & Q functions

Recursive Bellman expansion (from definition of Q)

(Expected) return from t = 0

$$Q^*(s,a) = \mathop{\mathbb{E}}_{\substack{a_t \sim \pi^*(\cdot|s_t) \\ s_{t+1} \sim p(\cdot|s_t,a_t)}} \left[\sum_{t \geq 0} \gamma^t r(s_t, a_t) \mid s_0 = s, a_0 = a \right]$$

Recursive Bellman expansion (from definition of Q)

(Expected) return from t = 0

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$$= \gamma^0 r(s,a) + \mathop{\mathbb{E}}_{s' \sim p(\cdot|s,a)} \left[\gamma \mathop{\mathbb{E}}_{\substack{a_t \sim \pi^*(\cdot|s_t) \\ s_{t+1} \sim p(\cdot|s_t,a_t)}} \left[\sum_{t \geq 1} \gamma^{t-1} r(s_t, a_t) \mid s_1 = s' \right] \right]$$

$$= r(s,a) + \gamma \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} [V^*(s')]$$

$$= \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} [r(s,a) + \gamma V^*(s')]$$

(Reward at t = 0) + gamma * (Return from expected state at t=1)

- Equations relating optimal quantities

$$\boxed{V^*(s) = \max_a Q^*(s,a)}$$

$$\boxed{\pi^*(s) = \arg\max_a Q^*(s,a)}$$

- **Recursive Bellman optimality equation**

Bellman Optimality Equations

- Equations relating optimal quantities

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- **Recursive Bellman optimality equation**

$$\begin{aligned}
Q^*(s,a) &= \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} \left[r(s,a) + \gamma V^*(s) \right] \\
&= \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^*(s) \right] \\
&= \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma \max_a Q^*(s',a') \right]
\end{aligned}$$

Bellman Optimality Equations

- Equations relating optimal quantities

$$\boxed{V^*(s) = \max_a Q^*(s,a)} \qquad \boxed{\pi^*(s) = \arg\max_a Q^*(s,a)}$$

- **Recursive Bellman optimality equation**

$$\begin{aligned}
Q^*(s,a) &= \mathop{\mathbb{E}}_{s'\sim p(s'|s,a)} \left[r\left(s,a\right) + \gamma V^*(s)\right] \\
&= \sum_{s'} p\left(s'|s,a\right)\left[r\left(s,a\right) + \gamma V^*(s)\right] \\
&= \sum_{s'} p\left(s'|s,a\right)\left[r\left(s,a\right) + \gamma \max_a Q^*(s',a')\right]
\end{aligned}$$

$$V^*(s) = \max_a \sum_{s'} p\left(s'|s,a\right)\left[r(s,a) + \gamma V^*\left(s'\right)\right]$$

Bellman Optimality Equations

Bellman Optimality Equations

Bellman equation:

$$V^*(s) = \max_a \sum_{s'} p\left(s'|s,a\right)\left[r(s,a) + \gamma V^*\left(s'\right)\right]$$

Bellman equation:

$$V^*(s) = \max_a \underbrace{\sum_{s'} p(s'|s,a)}\left[r(s,a) + \gamma V^*(s')\right]$$

Value of a given state

If we act optimally

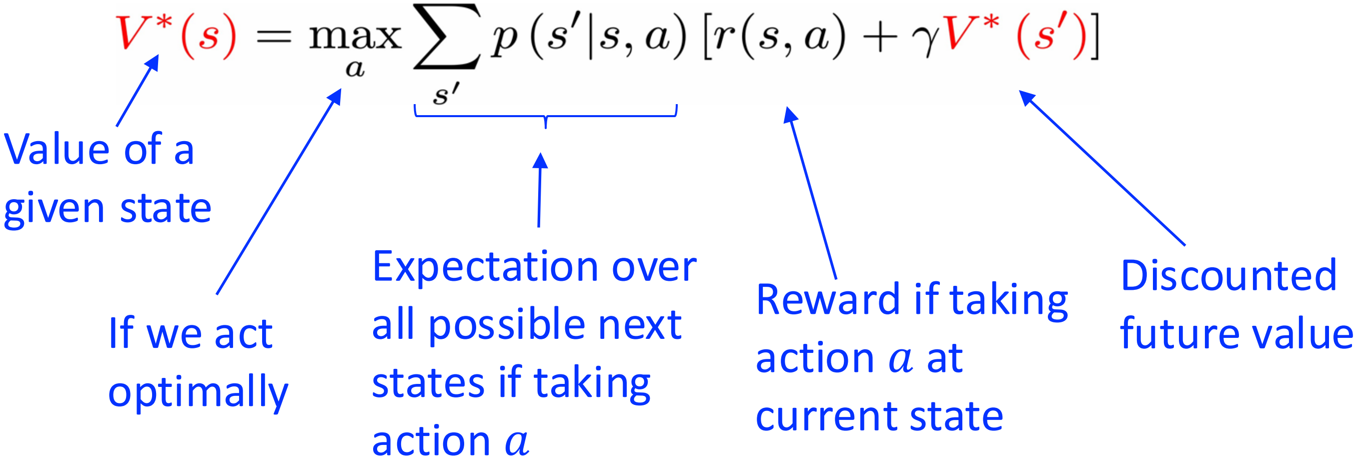
Expectation over all possible next states if taking action a

Reward if taking action a at current state

Discounted future value

Bellman Optimality Equations

Bellman equation:



$$V^*(s) = \max_a \underbrace{\sum_{s'} p(s'|s,a)} [r(s,a) + \gamma V^*(s')]$$

Value of a given state

If we act optimally

Expectation over all possible next states if taking action a

Reward if taking action a at current state

Discounted future value

Bellman equation: the optimal value of a state equals to the immediate reward plus discounted future rewards, when acting optimally

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Can we use this equation to construct a learning algorithm of V*?

Bellman Optimality Equations

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Bellman equation:

$$V^*(s) = \max_a \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma V^*(s')\right]$$

Goal: Learn a value function V that correctly maps states to optimal values.

Bellman equation:

$$V^*(s) = \max_a \sum_{s'} p\left(s'|s,a\right)\left[r(s,a) + \gamma V^*\left(s'\right)\right]$$

Goal: Learn a value function V that correctly maps states to optimal values.

Facts:

- If a value function V is correct, then this equation should hold exactly.

Bellman Optimality Equations

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- If a value function V is correct, then this equation should hold exactly.
- If the value function is incorrect, we can use this equation to update the value estimate.

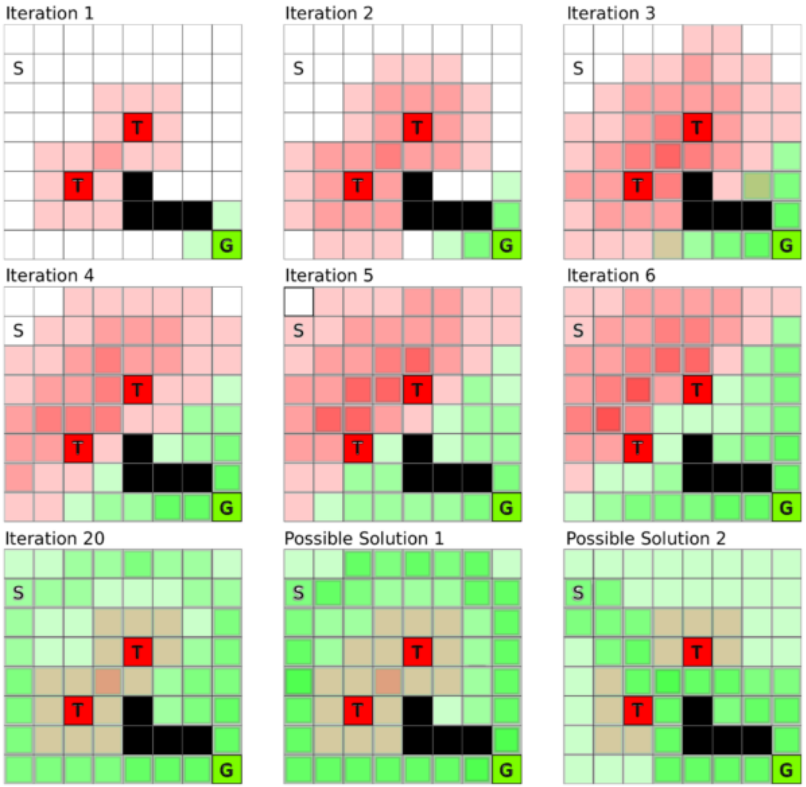
$$\boxed{V^{i+1}(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma V^i(s')\right]}$$

Value Iteration

$$V^{i+1}(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma V^i(s')\right]$$

```
Initialize Value Function table
For each iteration $i$:
-   For each state $s$:
    -   For each action $a$:
        -   Get reward $r(s, a)$
        -   For each possible future states $s$’:
            -   Get current $V(s’)$ from table
        -   Compute the expectation term
    -   Select the highest future value for $a$
    -   Update new $V(s)$
```

This algorithm looks familiar ...
It’s dynamic programming!



https://developer.nvidia.com/blog/deep-learning-nutshell-reinforcement-learning/

Value Iteration

Value Iteration Update:

$$V^{i+1}(s) \leftarrow \max_a \sum_{s'} p(s'|s,a)\left[r(s,a) + \gamma V^i(s')\right]$$

Q-Iteration Update:

$$Q^{i+1}(s,a) \leftarrow \sum_{s'} p\left(s'|s,a\right)\left[r\left(s,a\right) + \gamma \max_{a'} Q^i(s',a')\right]$$

Given a learned Q function, we can derive the *optimal policy*:

$$\pi(s) = argmax_a Q(s,a)$$

Value Iteration

Algorithm: Value Iteration

- Initialize values of all states to arbitrary values, e.g., all 0’s.
- While not converged:
  - For each state: $V^{i+1}(s) \leftarrow \max_{a} \sum_{s'} p(s'|s,a) \left[r(s,a) + \gamma V^{i}(s') \right]$
- Repeat until convergence (no change in values)

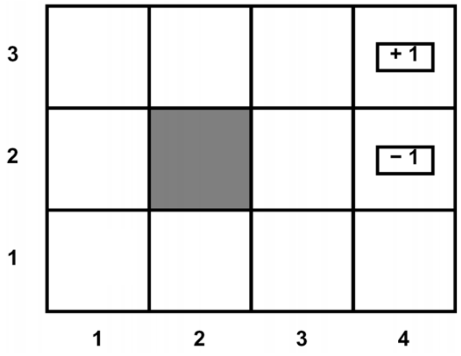
$$V^0 \rightarrow V^1 \rightarrow V^2 \rightarrow \quad \cdots \rightarrow V^i \rightarrow \ldots \quad \rightarrow V^*$$

Q: What’s the time complexity per iteration?

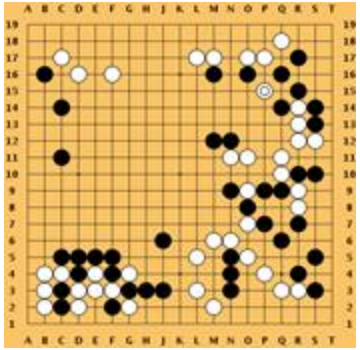
Time complexity per iteration $O(|\mathcal{S}|^2|\mathcal{A}|)$

Value iteration is almost never used in practice!

Time complexity per iteration $O(|\mathcal{S}|^2|\mathcal{A}|)$



$|S| = 11, |A| = 4$



$|S| \cong 3^{361}, |A| \cong 361$

$|S| \cong ?, |A| = ?$

Can't iterate over all (s, a) pairs -> need approximation!

We also don't know the transition function (model) -> need a *model-free* method!

Q-Learning

- We’d like to do Q-value updates to each Q-state:

$$Q'(s_t, a_t) \cong \sum_{s'} T(s_{t+1}|s_t, a_t)[r_t + \gamma \max_a Q(s_{t+1}, a)]$$

  - But can’t compute this update without knowing the transition function and enumerate all possible next states s’!

- Instead, *approximate* the expectation (sum over next states) with (lots of) experience samples
  - Take an action in the environment following *policy* $\mathrm{argmax}_{\mathrm{a}} Q(s, a)$
  - receive a sample transition (s_t, a_t, r_t, s_{t+1})
  - This sample suggests: $Q(s_t, a_t) \cong r_t + \gamma \max_a Q(s_{t+1}, a)$
  - Keep a running average to approximate the expectation:

$$Q'(s_t, a_t) = (1 - \alpha)Q(s_t, a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1}, a)]$$

Old estimates

New estimates

Q-Learning

Approximate the expectation (sum over next states) with (lots of) experience samples

- Take an action in the environment following *policy* $\mathrm{argmax}_\mathrm{a} Q(s, a)$
- receive a sample transition (s_t, a_t, r_t, s_{t+1})
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$$Q'(s_t, a_t) = (1 - \alpha) Q(s_t, a_t) + \alpha [r_t + \gamma \max_a Q(s_{t+1}, a)]$$

- We can now learn Q values without having access to a transition model
- Getting experience data through interaction instead of assuming access to all states: more practical in real-world situation (e.g., robots learning through trial-and-error)
- Still need to represent all (s, a) pairs in a Q value table!

Q-Learning

Idea: represent the Q value table as a **parametric function** $Q_\theta(s,a)$!

How do we learn the function? We need a **loss metric**!

$$\begin{aligned} Q'(s_t,a_t) &= (1-\alpha)Q(s_t,a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1},a)] \\ &= Q(s_t,a_t) + \alpha(r_t + \gamma \max_a Q(s_{t+1},a) - Q(s_t,a_t)) \end{aligned}$$

Now, at optimum, $Q(s_t,a_t) = Q'(s_t,a_t) = Q^*(s_t,a_t)$; This gives us:

$$0 = 0 + \alpha(r_t + \gamma \max_a Q(s_{t+1},a) - Q(s_t,a_t))$$

Learning problem:

$$\text{argmin}_\theta || \underbrace{r_t + \gamma \max_a Q_\theta(s_{t+1},a)}_{\text{Target Q value}} - Q_\theta(s_t,a_t)) ||$$

How to model Q?

Deep Q-Learning

- **Q-Learning** with **linear function approximators**

$$Q(s, a; w, b) = w_a^\top s + b_a$$

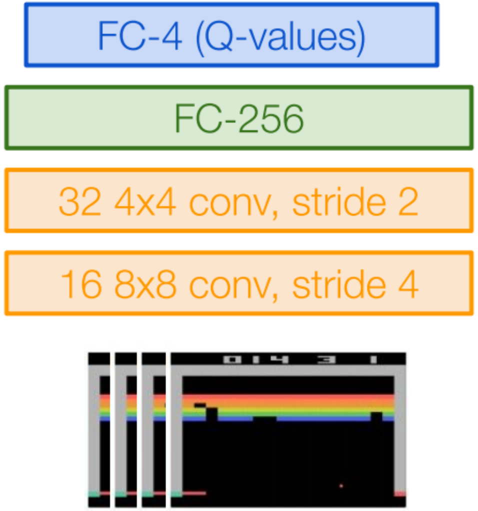
  - Has some theoretical guarantees

- **Deep Q-Learning**: Fit a **deep Q-Network**

$$Q(s, a; \theta)$$

  - Works well in practice
  - Q-Network can take arbitrary input (e.g. RGB images)
  - Assume discrete action space (e.g., left, right)

Value per action dim



FC-4 (Q-values)

FC-256

32 4x4 conv, stride 2

16 8x8 conv, stride 4

Deep Q-Learning

- Assume we have collected a dataset:

$$\{(s, a, s', r)_i\}_{i=1}^{N}$$

- We want a Q-function that satisfies bellman optimality (Q-value)

$$\boxed{Q^*(s,a) = \mathop{\mathbb{E}}_{s' \sim p(s'|s,a)} \left[r\,(s,a) + \gamma \max_{a'} Q^*(s',a') \right]}$$

- Loss for a single data point:

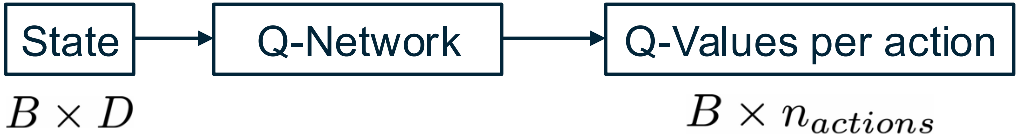
$$\text{MSE Loss} := \Big(\underbrace{Q_{new}(s,a)}_{\text{Predicted Q-Value}} - \underbrace{(r + \gamma \max_{a} Q_{old}(s',a))}_{\text{Target Q-Value}}\Big)^2$$

Deep Q-Learning

Deep Q-Learning

- Minibatch of $\{(s,a,s',r)_i\}_{i=1}^{B}$

- Forward pass:



State → Q-Network → Q-Values per action

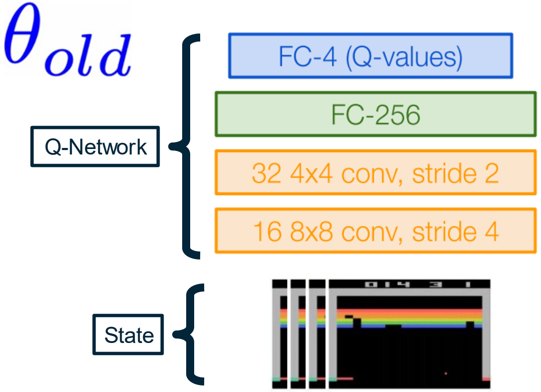
$B \times D$ $\qquad$ $B \times n_{actions}$

- Compute loss:

$$\Big(\underbrace{Q_{new}(s,a)}_{\theta_{new}} - (r + \gamma \max_{a} \underbrace{Q_{old}(s',a)}_{\theta_{old}})\Big)^2$$

- Backward pass:

$$\frac{\partial Loss}{\partial \theta_{new}}$$



$$\text{MSE Loss} := \left(Q_{new}(s,a) - (r + \max_a Q_{old}(s',a))\right)^2$$

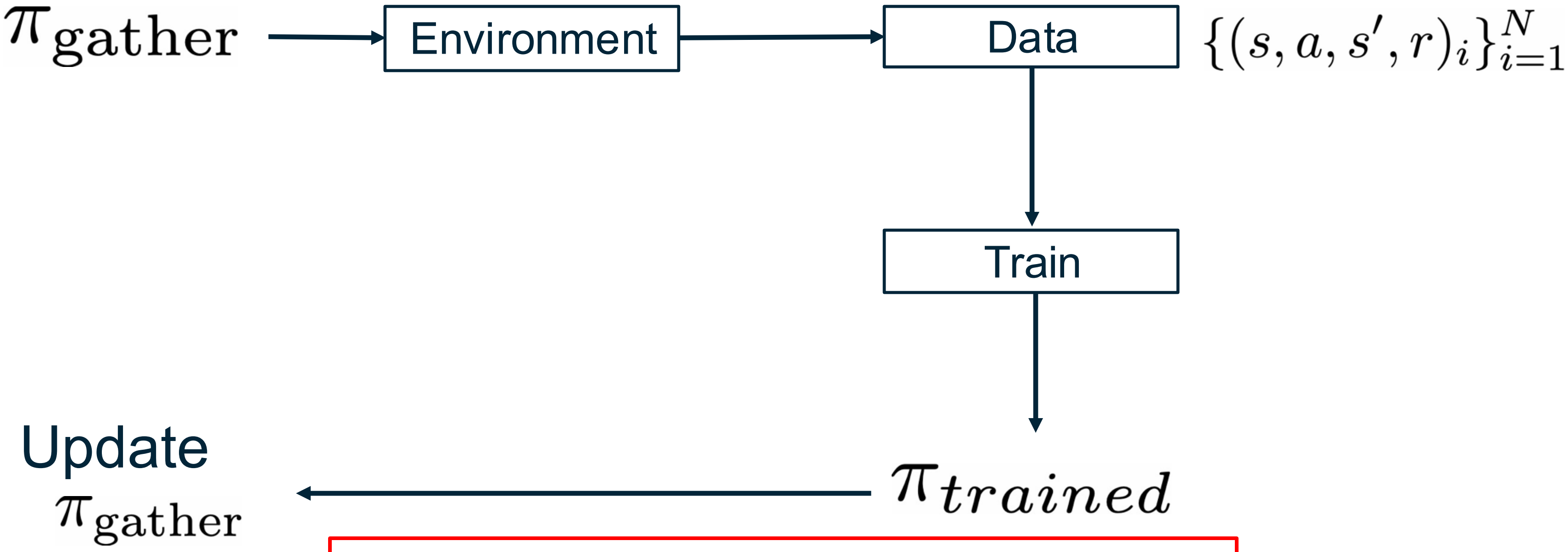
- In practice, for stability:
  - Freeze Q_{old} and update Q_{new} parameters
  - Set $Q_{old} \leftarrow Q_{new}$ at regular intervals or update as running average
    - $\theta_{old} = \beta\theta_{old} + (1-\beta)\theta_{new}$

Deep Q-Learning

How to gather experience?

$$\{(s, a, s', r)_i\}_{i=1}^{N}$$

This is why RL is hard



Challenge 1: Exploration vs Exploitation

Challenge 2: Non iid, highly correlated data

How to gather experience?

- What should π_{gather} be?
  - Greedy? -> no exploration, always choose the most confident action

    $$\arg\max_a Q(s, a; \theta)$$
- An exploration strategy:
  - ϵ-greedy

$$a_t = \begin{cases} \arg\max_a Q(s, a) & \text{with probability } 1 - \epsilon \\ \text{random action} & \text{with probability } \epsilon \end{cases}$$

Exploration Problem

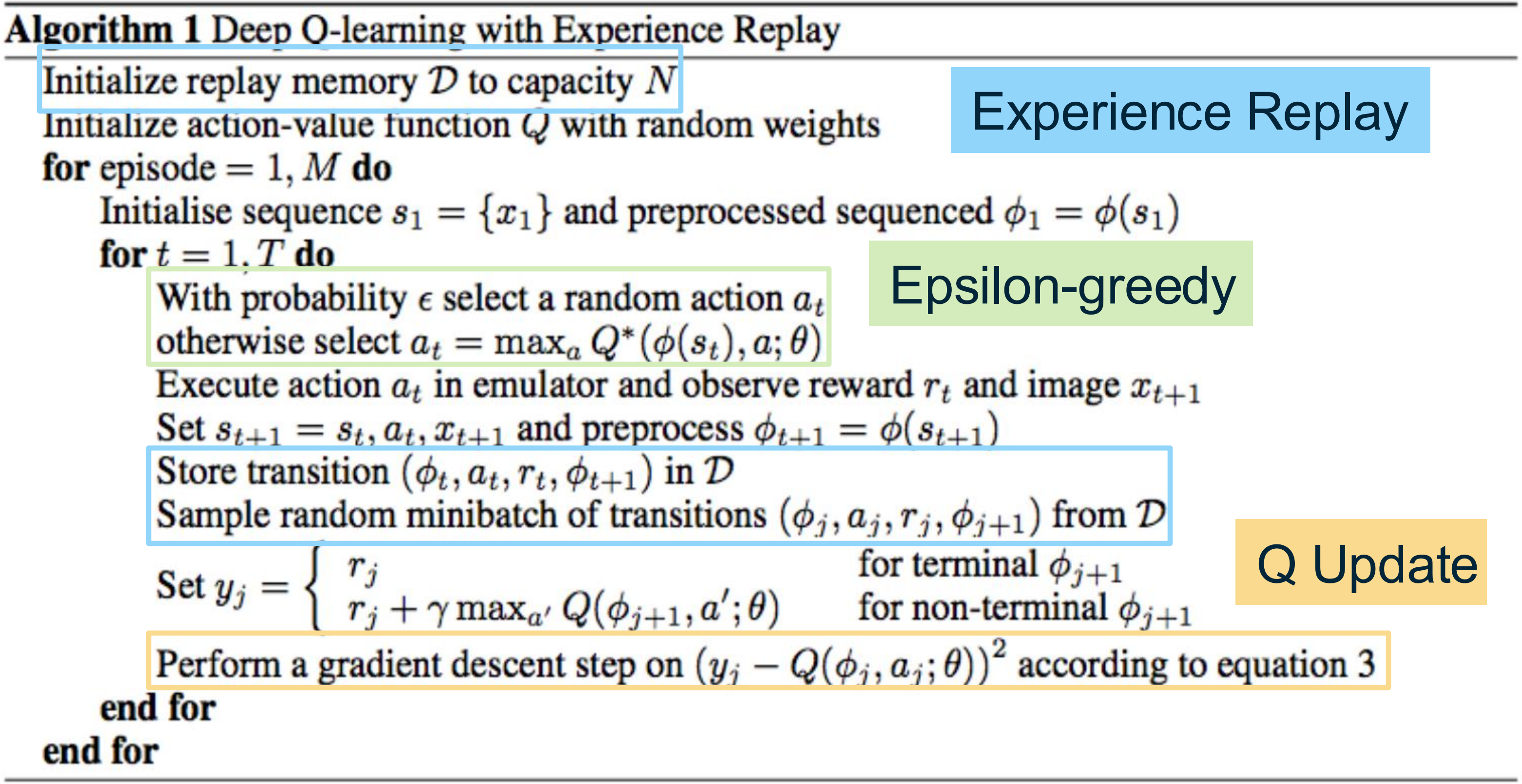
- Samples are correlated => high variance gradients => **inefficient learning**

- Current Q-network parameters determines next training samples => can lead to **bad feedback loops**
  - e.g. if maximizing action is to move right, training samples will be dominated by samples going right, may fall into local minima

Correlated Data Problem

Experience Replay

- Correlated data: addressed by using **experience replay**
  - A replay buffer stores transitions (s, a, s', r)
  - Continually update replay buffer as game (experience) episodes are played, older samples discarded
  - Train Q-network on random minibatches of transitions from the replay memory, instead of consecutive samples
- Larger the buffer, lower the correlation



```
Algorithm 1 Deep Q-learning with Experience Replay
Initialize replay memory $\mathcal{D}$ to capacity $N$
Initialize action-value function $Q$ with random weights
for episode $= 1, M$ do
    Initialise sequence $s_1 = \{x_1\}$ and preprocessed sequenced $\phi_1 = \phi(s_1)$
    for $t = 1, T$ do
        With probability $\epsilon$ select a random action $a_t$
        otherwise select $a_t = \max_a Q^*(\phi(s_t), a; \theta)$
        Execute action $a_t$ in emulator and observe reward $r_t$ and image $x_{t+1}$
        Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in $\mathcal{D}$
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from $\mathcal{D}$
        Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(\phi_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
        Perform a gradient descent step on $(y_j - Q(\phi_j, a_j; \theta))^2$ according to equation 3
    end for
end for
```

Experience Replay

Epsilon-greedy

Q Update

Deep Q-Learning Algorithm

Atari Games

- **Objective**: Complete the game with the highest score
- **State**: Raw pixel inputs of the game state
- **Action**: Game controls e.g. Left, Right, Up, Down
- **Reward**: Score increase/decrease at each time step

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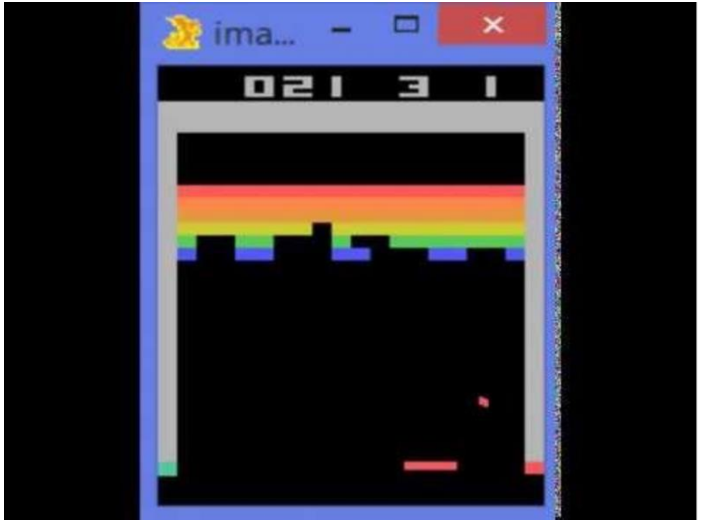
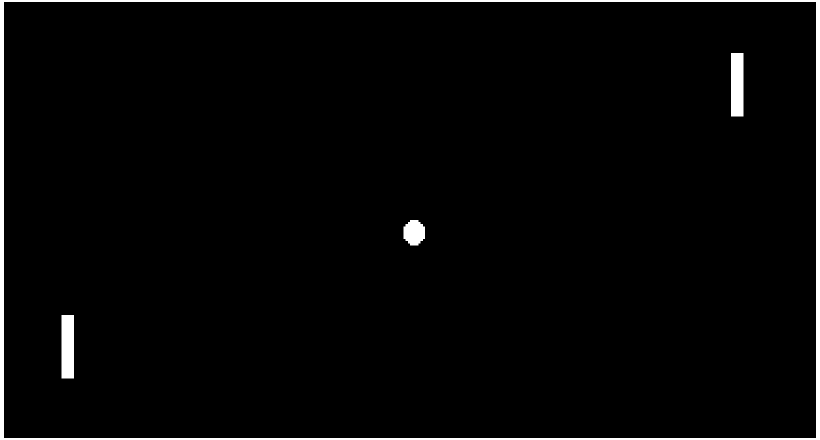
Atari Games





https://www.youtube.com/watch?v=V1eYniJ0Rnk

Different RL Paradigms

- **Value-based RL**
  - (Deep) Q-Learning, approximating $Q^*(s, a)$ with a deep Q-network
- **Policy-based RL**
  - Directly approximate optimal policy π^* with a parametrized policy π_θ^*
- **Model-based RL**
  - Approximate transition function $T(s', a, s)$ and reward function $\mathcal{R}(s, a)$
  - Plan by looking ahead in the (approx.) future!

Today, we saw

- **MDPs**: Theoretical framework underlying RL, solving MDPs
- **Policy**: How an agents acts at states
- **Value function (Utility)**: How good is a particular state or state-action pair?

- **Solving an MDP with known rewards/transition**
  - **Value Iteration:** Bellman update to state value estimates
  - **Q-Value Iteration:** Bellman update to (state, action) value estimates

- **Deep Q Learning**
  - Model Q value function with a deep NN!

Summary: MDP Algorithms

Next Time: RL continued --- Policy Gradient and Actor-Critic